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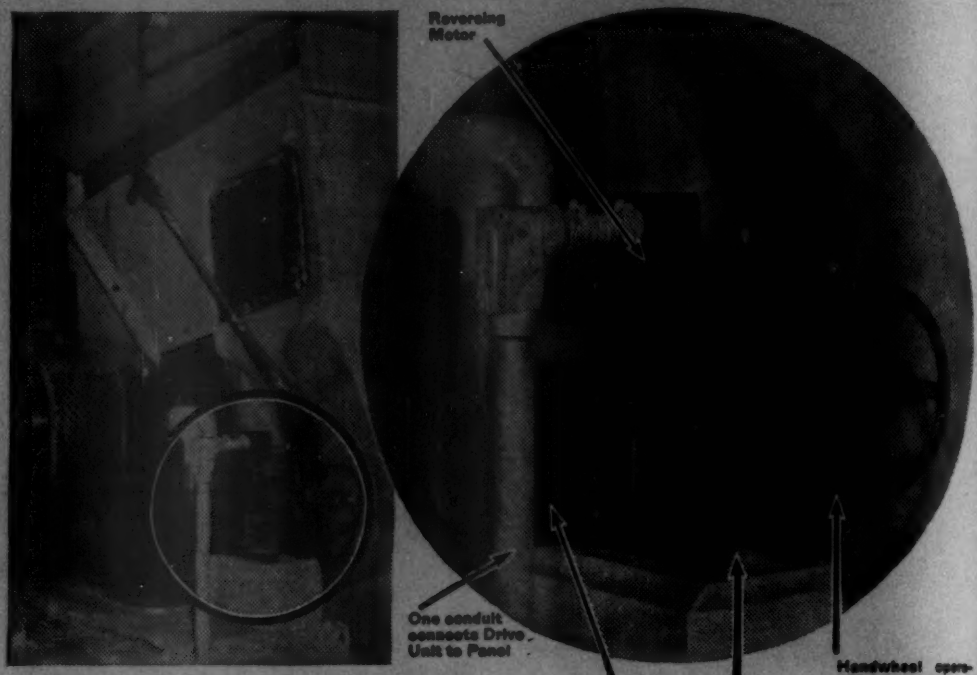
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Journal
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Vol. 242 DECEMBER, 1946 No. 6

PROGRESS OF THE BIOCHEMICAL RESEARCH FOUNDATION (1946).

BY

ELLICE McDONALD, M.D.,

Director.

In the past year we have finished our work with the Manhattan Engineering District upon effects of emanation of atomic forces. This will be published by the Manhattan Engineering District in a technical report.

In agreeing to this work, we made the reservation that such of our workers who were not occupied in the contract should be free to do research upon associated problems, provided it did not interfere with the contract work. This limited our study and compelled us to adapt our efforts to such neutron dosages as were required by them. These were smaller amounts of neutron dosages and these were to be produced daily. The interest of the Manhattan Engineering District was to find the results of neutrons upon the white blood cells of animals and to expose these animals day after day.

Our interest was wider and we were not so much concerned with the end results of neutron radiation as with an explanation of how neutron radiation produced its effects. We were interested in the *mechanism* of the reaction because only in that way could injury be prevented and therapeutic efforts intelligently applied. Our concern was not only to prevent the impact of radiation by screening with walls and water to keep the neutrons from reaching the operator, but also to determine the *mechanism* of the action: for only with this knowledge can remedial and curative measures be applied after the patient has absorbed the neutrons.

In these days of the atomic bomb and the possibility of atomic energy being applied to industrial organization, the study of its forces seems very pertinent, so we attempted to study the effects of neutrons, the chief of the forces of atomic fission, in many different ways, adapting

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our study to the dosage required by the Manhattan Engineering District project. It may be of interest that our cyclotron ran daily for one year without missing a day, a record for continuous performance we believe; anyone with any experience in running a cyclotron with its aberrations and temperamentality will know what effort and foresight that required.

Our research which was of the nature of an exploration led us into many different places. The field was new and it was impossible, in the beginning, to say in which direction the most promising results lay. It was like trying to find a pass through a mountain range and sending out a number of explorers with the hope that one would find a way through. We were fortunate in having workers with a background of biochemistry and biology, so that we did not have to improvise these. Our cyclotron is the only one which was built as an adjunct to a biochemical organization: all others were created by physicists.

We had many negative results, but these had to be explored to find whether they were negative or not. In searching for the mountain passes, all the explorers could not be successful: some must return with negative results, and so with us in our exploration into the effects of neutrons we wanted to find the best way.

These results, positive and negative, will be published soon in a book of 200 pages entitled "Neutron Effects on Animals." There will be 20 chapters, 31 photographs, 38 graphs, and 31 tables. The volume will describe neutron effects on enzymes, blood, bone marrow, electrophoretic studies on the proteins of the blood plasma, fluorescent spectroscopy, blood sedimentation, tissue changes, physiological response, resistance to infection, and other phases. Our hope is to detail methods of study for future investigators.

Radiation effects have interested the Biochemical Research Foundation for 15 years. At first it was study in regard to X-rays and ultraviolet light effects, but for the past eight years it has been the study of forces and material produced by the cyclotron.

Although radiation is grouped under one term, radiations are varied in character; visible light, ultraviolet, X-rays, gamma rays, neutrons, etc., may all be emanations or radiations, but that they have the same mechanism for producing their results is a conclusion so far-fetched and erroneous as to be reached only because of the mystery of the subject. Neutron bombardment and X-radiation may produce somewhat similar results as shown in white blood cell counts and dead cells in pathologic study, but it by no means follows that they achieve these results by a similar mechanism. The cause in both cases may be radiation, and the end results—destruction of cells—may be microscopically similar in the dead tissue, but the intermediate mechanism by which these results are attained may be quite different. It may be that neutron and X-ray effects do have a similar mechanism in producing their destructive results, but there is at present no proof that this is so.

Indeed, we have an example which shows certain lethal radiations have a mechanism which is probably quite different from X-rays. Certain short wave lengths of ultraviolet light are extremely lethal to cells when brought into their proximity and act in an apparently different mechanism from X-rays.

In 1934, we set up an all-quartz microscope and made a monochromator attachment so that photographs could be taken of the effects of various wave lengths of ultraviolet light measured in Ångstrom units. We also made moving pictures of the killing of motile cells such as euglena and paramecia in order to study the mechanism of their destruction. Cessation of their movement showed that they were killed.

We found that certain wave lengths of short ultraviolet light were more destructive than others and this because the lethal wave lengths were absorbed by the nucleus of the cell. The absorption of the quanta may be seen by the "blackening" of the nucleus of the euglena. Until the neighborhood of wave length 2804 Å, there was no absorption of the quanta of ultraviolet light and no killing resulted. When the greatest blackening or absorption of quanta by the nucleus occurred at the line 2536 Å, the cells were killed within one or two seconds. Yet the euglena treated by X-rays required 20,000 r in 24 hours in order to produce the same lethal results. Lower than 2536 Å the absorption was not so great (1).

The line 2536 Å of ultraviolet light is the *most lethal wave length yet discovered*. It is hundreds of times more lethal to cells than high voltage X-rays. Its disadvantage is the lack of penetration through tissues.

The partial mechanism of this wave length is the absorption of the quanta by the nucleic acid of the nucleus of the cell since nucleic acid has an ultraviolet spectrographic peak, an extinction coefficient, in the neighborhood of 2536 Å: it is a resonance effect and a similarity of absorption coefficient. These shorter ultraviolet wave lengths apparently break down certain of the molecular bonds, particularly the polypeptide linkages which are so frequent in the amino acids.

That the same course is followed in X-ray effects seems very doubtful for there is little or no "blackening" of the cell nucleus by X-rays. It may be possible that the killing effect of X-rays is due to secondary radiation with production of wave lengths at a lower wave length, a Compton-Einstein effect, in the neighborhood of 2536 Å, but there is little or no evidence as to this. When the enormously greater energy delivered by X-rays is considered, the comparatively slighter lethal effect is surprising, so it must be concluded that only part of its energy is effective and some explanation of this fact is required.

Similar experiments have been done by us upon nucleated red blood cells of fowls which showed that the maximum absorption of the monochromatic light was at 2654 Å (2): a small deviation from the previous shorter wave length, 2536 Å.

As far as X-rays and neutrons go, there is little correlation between their mechanism upon the cells and that of the shorter ultraviolet rays. If the theory of secondary radiation at a wave length comparable to the killing wave length of ultraviolet is disproved, some other explanation must be sought.

The vital system is not as easily explained as simple chemical reactions, but life is after all a chemical phenomenon and must be considered upon that basis. The large number of catalysts, such as enzymes, hormones and vitamins, complicate the explanation. These catalysts are specific and effective in enormously minute amounts, so that their effect is often overlooked in a general view of the vital system. No real conclusion as to effects of forces or substances can be really clear unless there is some elucidation of the mechanism involved rather than an explanation based solely on cause and end-result without consideration of the mechanism.

All chemical exchanges exist through the giving or taking or sharing of electrons, so that this must be considered as fundamental. There are three types of chemical systems. First, there are true solutions, that is strong acid and strong base, where the reactions are complete in one direction and reversible only to a perceptible degree, of the type $H_2SO_4 + 2NaOH = Na_2SO_4 + 2H_2O$, but this obviously cannot be the one we are concerned with in systems within the mammalian body.

Type two, another explanation of vital reactions, and it was in its time quite plausibly advanced by Arrhenius, is that of weak acid and weak base in which the equilibrium is reached while there are still present appreciable amounts of free acid and free base as well as neutral salt, the quantities depending, according to the law of mass action, upon the relative amounts in which the two reagents are present in the mixture. Attractive as this explanation is of the chemical action, particularly of immunological reactions, it has not been found acceptable, although the explanations of Pauling and others more recently suggest that character.

Colloid systems, type three, are treacherous, for the more they are dispersed, the more closely they follow the chemical form of weak acid and weak base and approach combination in stoichiometric proportion. The vital system is not only a colloid system but it is a series of colloid systems within colloid systems. It exists in a state of balance or equilibrium (not equilibrium in the chemical sense), and this balance is necessary to the continuance of function. If it is out of balance, the system will cease to function.

The cell is the unit of life, but the cell is in an environment and the environment is part of the colloidal cellular system. The cell and its environment are one. One of the difficulties of biochemical research is that it is impossible, except perhaps in the blood, to remove a cell from its environment for study. A fish removed from its watery environment

is no longer a fish that leaps and plunges but a dead thing; so also is a cell removed from its environment.

Lord Kelvin said in regard to scientific theories: "If I can make a model of it I can try to understand it and if I cannot make a model of it it is difficult for me to understand it." It is possible to make a model of a vital system considering the cell as its unit, but this model is no more accurate than the model of the atom with fixed electrons of Bohr and Lewis in 1918, which has since been developed into the orbits of moving electrons and all their influence upon nuclear physics. Both the living cell and the atom are dynamic systems. Such a model (Fig. 1) for a dynamic vital system can no more be right than a model of the atom with fixed electrons, but it is a step toward understanding and explanation. It merely is a diagrammatic representation for convenience of thought and clarity of consideration.

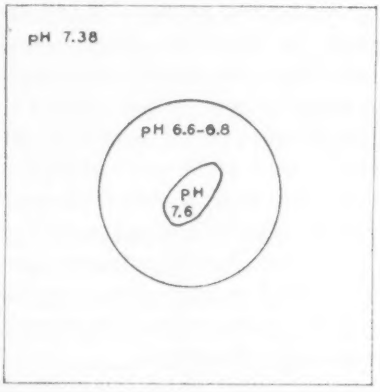

FIG. 1. Diagrammatic model of the cell and its environment.

The cell protoplasm is toward the acid side, surrounded by a boundary or membrane; the cell nucleus is alkaline; the environment, the blood plasma, is slightly alkaline. These changes in acid-base equilibrium (pH) may seem small to the chemist, but they loom large to the student of vital systems where small changes in pH are of great importance, for upon that as one of the factors depends the continuance of the vital equilibrium or balance, the *milieu interne* of Claude Bernard.

Thus, in attempting to explain the action of neutrons upon vital systems, one question must be answered. In what part of the vital system does the effect take place? Is it on the nucleus, the protoplasm, the cell membrane or the environment? If that can be answered the area of research is lessened. In the action of short ultraviolet light, the effect is apparently upon the nucleus. In X-rays' effects, there is no proof, but it is significant that the effect of X-rays is greater upon those cells having greater amount of cell nuclear elements. This is shown

particularly in growing, immature tissue and in certain cancers which are spoken of as "radiosensitive" in that they are more easily affected by X-rays, and have, as a rule, a larger amount of nuclear tissue containing nucleic acid than normal. Apart from this, there is little evidence as to the site of X-ray effect.

The most significant fact in neutron bombardment is its effect upon growing or immature tissue: this tissue has the largest amount of nuclear elements. This is most marked in the testes, the lymph tissue and the white blood corpuscles. The latter may be considered extremely young tissues, for in man they are renewed at least every twenty-four hours. The normal white blood corpuscle count of man is from 5000 to 8000 white cells per cubic millimeter. If the amount of blood in a man of 145 pounds is taken as 6 liters, then 30 billion white blood corpuscles are renewed every day. This seems incredible, yet many things connected with the human body seem incredible.

The number of lymphocytes present in the circulating blood of the cat has been estimated by Sanders, Florey and Barnes (3) at 1250 million. This number, they calculated, was replaced from one-half to almost 3 times every twenty-four hours. For rabbits the daily output of lymphocytes by the thoracic duct was 4×10^9 and the lymphocyte population was replaced with fresh cells about five times daily. The white blood corpuscles have such a short life and are so frequently renewed that they must be considered extremely young, fragile and immature tissues. They must also be present, owing to their short life and quick replacement, in all stages of very rapid growth and development, since their total life is not more than twenty-four hours. They are truly impermanent and ephemeral.

The source of the white blood corpuscles is believed to be in the bone marrow, the spleen, and lymphatic tissue. By giving a sufficient dose of neutrons it is possible to abolish all white blood corpuscles from the blood, and it may even be possible after a large dose of neutrons to find bone marrow without a single leukocyte. Use is made of this extraordinary fact by taking the level of the white blood cell count as a measure of the amount of injury produced by the neutrons. It is a test easy to do and familiar to laboratory technicians. It is by no means exact because it is influenced by other factors: resistance, age, etc.

The reduction and even disappearance of the white blood cells do not seem to be dependent upon direct killing of these cells, for untreated white blood cells, introduced after exposure to neutrons, live for some time and are not directly killed by repeated radiation. *Something happens which prevents the continued production of these short-lived, easily reconstructed cells.* It is as if a population ceased to be fertile and produced no offspring: they live their life and at the end are not any more.

Such a result in such short-lived cells is significant of the disruption of the processes of their reproduction. If the dose of neutrons is not

so great, recovery may take place in time. There is a possibility of the recrudescence of the specific vital factor which is necessary to their continuance.

In consideration of the mechanism of neutron action, it is futile to speak in terms of ionization when there is no idea of where in the vital system that ionization exists. Is it in the nucleus, the cell, the cell boundary, or the membrane? It is true that the greater the amount of neutrons applied the greater the results, but such an amount of energy, hundreds of times that of the short ultraviolet line 2536 Å, has comparatively less efficiency as a lethal instrument.

A word should be said about the variability of experimental animals. Chemists and physicists without much experience in animal experimentation are apt to think of them as exact systems, similar to those of the sciences in which they work and in which there are definite quantitative measurements. Not only are different species of animals different in their reactions, but different strains of the same species vary in their reactions. There are strains of mice which have naturally occurring cancer and strains that are cancer-resistant. There are strains of rats that are resistant to transplanted cancer and those that are not. The same is true of many other qualities: age, diet, and even environment may alter their reactions.

Then too, the vital system is a dynamic system: its chief purpose is life and it goes on living. Alterations in one or other phase will upset the system, but this cannot be explained in terms of such exactitude as exists in chemical and physical experiments. The distinguishing phases of vital systems are that they are provided with the means for the rapid attainment of equilibrium or vital balance and that they are equipped with highly specific catalysts, enzymes, hormones and vitamins which, in selected chemical reactions, determine the course of the metabolic processes. These catalysts in minute amounts are necessary for the continuance of life and more particularly for the synthesis of rapidly growing cells such as the lymphocytes which disappear after severe neutron bombardment.

In neutron effects, on account of the severity of the result and the quickness of action, vitamins and hormones are less likely to be the deficiency because the effect of their lack may not be shown as quickly as the other neutron effects. The neutron effect may be a two-stage or even a three-stage effect; the altering of these substances may prevent the continuance of the metabolic processes. An example of this is the interference with sulfur metabolism so necessary to growing cells, the chief source of which is methionine, biotin, and thiamine.

The effects of neutrons may be upon one of the enzymes or it may be on the enzyme activator. The substrate may also be affected.

The enzymes, which are all proteins, are dependent upon their structural elements and there is an active catalytic center to involve the

combination of the substrate with the active center of the enzyme to form a dissociable enzyme-substrate complex. The substrate is activated and this activated substrate molecule then undergoes rapid reaction with another substance. Both the catalytic activity and the specificity of its actions depend upon the nature of the protein.

Attached to the proteins are prosthetic groups, often of low molecular weight, which combine reversibly with specific proteins and so undergo rapid oxidation-reduction reactions. The prosthetic group is usually a partner in the chemical reaction catalyzed by the enzyme. The active catalytic center is made up of the structural elements in the protein with which the prosthetic group combines. The enzymes, catalase and peroxidase, are each a complex protein in which a prosthetic group, hemin, is linked to a specific protein. In our research it has been found that after neutron bombardment, catalase is not affected while peroxidase is much reduced, so that in this case at least the effect is not upon the prosthetic group although in the case of *d*-amino acid oxidase, the inactivation occurs in the prosthetic group, alloxine adenine nucleotide.

The prominent reactions of the cell are the processes of hydrolysis and condensation, and of oxidation-reduction. Organic substances may undergo other reactions such as amination and deamination, alkylation, etc., and these go on continuously and together. Characteristic of life is the large molecule such as protein, easily destroyed and easily reconstructed. Synthetic reactions require energy for their efforts and so they must be coupled with energy-producing reactions. The steady state, the vital balance, or *milieu interne*, represents a dynamic equilibrium in which enzymes are continuously catalyzing not only degradation and regeneration reactions, but also chemical reactions to make available for synthetic processes the energy acquired from the environment. The vital balance is determined, not only by the thermodynamic potentials, but also by the kinetics of the individual reactions catalyzed by the enzymes.

Very minute particles often play an extraordinarily large part in the metabolism of living tissue. For example, the growth hormone from the pituitary recently discovered by Evans of California is so powerful that ten gammas, ten-millionth of a gram, is sufficient to bring about a gain of weight of one gram a day in rats whose pituitary glands had been removed. This is only one example of the power which certain substances in very minute amounts have upon growth and reproduction of cells.

If the popular explanation of the physicists of absorption of quanta is to be made intelligent, it must be considered where, in the cell system, this absorption takes place. To take the homely metaphor of the automobile, if a bullet is fired from a rifle and hits the machine, the force is absorbed; but if it is absorbed on the fender, it does little harm and, if it is absorbed on the timer, the machine is dead. How much more

complicated and how much less understood is the dynamic cell system. If the quanta are absorbed on Evans' growth hormone, as quite well might be possible, then growth stops as do the white blood cells after neutron bombardment. It may be the destruction of the sensitive entity which prevents the growth and reduplication of cells. The mode of destruction might quite well be in a breakage of the weaker molecular linkages of the molecules concerned as seems to be the case in the effects from shorter ultraviolet light from 2654 Å to 2536 Å.

With the background of our previous work on the treatment of cells by shorter ultraviolet light and the absorption of these wave lengths by the nucleic acid of the cells, it was natural to begin studies in regard to enzymes concerned, particularly desoxyribonuclease, but this enzyme was shown not to be affected by neutron bombardment. Recent work by Dale, Meredith and Tweedie (4), however, has shown that in certain enzymes there is a protective or indirect action from additional substances in the solution capable of reacting with the intermediate products and so diverting some of the radiation to itself. The effect of X-rays on aqueous solutions of enzymes has been frequently interpreted in terms of the "quantum hit" theory without proper consideration of the indirect action. The effect depends upon the concentration and purity of the enzyme solutions and so shows reactions which would not be present in the vital systems owing to protective substances. This complicates the matter. Radiation produces a smaller effect with the added protective substances which are often too small to absorb radiation directly to any appreciable extent but the molecules take a share in the radiation product and thus the number of molecules in the main solute inactivated is reduced. So that the evidence of the effects of radiation upon pure enzyme solutes cannot now be taken at face value.

Recent and very interesting work by Mitchell upon X-rays (5) much along the same lines as our own, in regard to the effects of radiation upon the nucleic acid of the cell nucleus, comes to the conclusion, "Evaluation of the ionic efficiencies shows that significant disturbances of both carbohydrate and nucleic acid metabolism are produced by therapeutic doses of X and gamma radiations, probably by means of enzyme inactivation." After radiation in dividing cells he believes there is, due to mitotic inhibition, no significant increase in the thymonucleic acid synthesized by the nucleus. This histochemical evidence suggests that the small increase in nuclear absorption observed in some cases after radiation is due to an increase in pentose nucleotides either formed within the nucleus or diffused into it from the protoplasm. In our work upon neutrons, it has been considered that nucleic acid which is of large molecular weight, when broken into smaller molecular weight, pentose nucleotides, may diffuse when the larger molecular weight substance could not and that these nucleotides will inhibit the action of the enzyme ribonuclease to inhibit the synthesis of nucleic acid so necessary to

mitosis and cell division. This work is continuing and gives an interesting area of research into neutron mechanism.

Chemical processes, and life is a chemical phenomenon, are involved in the immediate effect of ionizing radiation. This is apparently true in regard to the turnover of desoxyribonucleic acid and Hevesy (6) has shown that after radiation with 1000 r of X-rays the turnover of phosphatide is also diminished from the nuclei and from the cytoplasm less so. This is of interest because of the importance of desoxyribonucleic acid phosphorus and phosphatide phosphorus in the nuclei of dividing cells.

One theory in regard to radiation effects is that there exists in the cell a special sensitive volume within which ionizations are biologically effective and that these account for subsequent changes. More than one ionization may be required to produce a biological effect but any ionization which occurs within the cell, but outside the sensitive volume, is ineffective. This view of the mode of action of radiation has become known as the "quantum hit" or target theory. Differences in sensitivity to radiation are explained by the chance distribution of ionization in the vital volume of the cell system.

An alternative hypothesis is that chemical or metabolic changes are produced in the cell (or in the environment?) by irradiation and that the biological results of physical as well as chemical agents may be explained upon the assumption that individual cells differ in the reactions and in the changes produced. The weakest (most radiosensitive) succumb first, then the less weak and the strongest (most radioresistant) the last of all.

The target theory cannot be made to fit all types of biologic response since by definition it made no allowance for adaptability in living organisms. The cell (and to speak of the cell must be to include the cell system with its environment) is not inert until it is dead and it is capable of recovery if it is not hurt too much: so long as it is alive it is capable of recovery. The types of response to radiation must be learned from observation under different biological conditions and the same cell may differ in its susceptibility to radiation under different conditions, as for example, dryness, metabolic activity, stage of growth and its age. There is a danger in attempting too much simplification by physical explanations in such a complex biological matter with its delicately poised and elaborate mechanism of the cell system.

The effect upon chromosomes in production of mutations in radiosensitive organisms, as Drosophila, is an argument against the target theory, since not all cells are equally susceptible to mutational effects of radiation but are also affected by temperature, nutrition, anesthesia and degree of germination. Environmental conditions alter the degree of response.

The number of chromosome breaks produced are proportional to the amount of the dose but independent of intensity (7, 8, 9), but neutrons

are more efficient in producing breaks than are X-rays. Mutation effects are dependent upon the wave length, independent of intensity, but general biological effects vary with alteration of the intensity and the wave length. These observations are explained on the hypothesis that a chromosome is broken by the passage through it of a single ionizing particle, but that it is necessary for the ionizing particle to be sufficiently densely ionizing for several ionizations to be produced near the chromosome. But this still gives no explanation as to what processes or chemical reactions are produced to cause the break in the chromosomes.

Radiation has a marked effect in interfering with cell proliferation and the dose which produces the first recognizable changes in cell proliferation is always small relative to the direct lethal dose for the same tissue. During development radiosensitivity decreases as the age increases, but the decrease is not necessarily progressive throughout development. It is a mistake to consider all cells in the body as of the same age. The frequently renewed white blood cells, for example, are young cells without much differentiation and differentiation reduces radiosensitivity.

Apart from a direct lethal effect, cells may be so injured by radiation as to be incapable of successful division and may thus either perish in attempting mitosis or produce non-viable daughter cells. In cancer it is obvious that, if sterilization of all potential dividing tumor cells could be achieved, their total destruction by radiation would be unnecessary, since the altered cells would gradually disappear due to their short life. Like weeds in a field of wheat, if prevented from multiplying without injury to the grain, the weeds will die out and it is unnecessary to kill both weeds and grain together.

Measurements of the biological action of X-rays and of γ-rays show that living cells vary in their response to irradiation. Differentiated cells are, in general, more radioresistant while dividing cells, immature cells, are more generally radiosensitive, and these are more sensitive at certain stages of their division cycle than at other stages. There is also variation in cells of a homogeneous population and this has been put forward as an evidence of individual variation and, on the contrary, as an expression of the relationship between the dose of radiation and the percentage of the cells which have received by chance a number of quantum hits to produce death, the "quantum hit" or target theory.

According to the first theory, when a group of cells is irradiated the radiant energy is equal in amount and is similarly distributed in each individual, whereas the capacity to resist the absorbed energy is different in different individuals. According to the second theory, the energy which is absorbed by cells or by specially vulnerable parts of cells is different in different individuals, whereas all the individuals have an equal capacity to resist a given injury.

The quantum hit theory assumes that the resistance of all individual cells to a given amount of absorbed energy is the same. The energy is absorbed in quanta and presumably the quantal absorption will have a random distribution amongst the different cells. Some have calculated that the number of quanta absorbed in the cell must be great if a lethal action is to occur and the quantum hit theory has been modified accordingly. The present form of the quantum hit theory is that a lethal number of quanta are absorbed in a particular "sensitive spot" in the cell and this causes its effect. It is as if a duck were shot with a large number of small shot and only one of a few struck the brain or another mortal part to produce death, while the other pellets were absorbed, but not to produce lethal effects.

Individual variation is a characteristic biological phenomenon and, no matter what care is taken to achieve homogeneity, the occurrence of individual variation can be proved in respect to every property of individual cells that is capable of measurement. It is reasonable to assume that in all living cells the resistance to injury varies. The difficulty in this assumption of variation is the absence of any plausible explanation as to how this variation, susceptibility or resistance to radiation is brought about.

The survival of some cells after a large dose of radiation may be due to the capacity to repair injuries which is different in different organisms or the radiation quanta may not affect certain of the cells in their sensitive spot. Usually organisms irradiated with a lethal dose survive for a period and then die. The tissues are capable of repairing the injury produced when the dose is small and so it may be counteracted, but when the dose is large enough, the repair processes produce a negligible effect upon the total amount of injury, so that at high intensities, the biological effect of radiation is independent of the intensity.

In all these considerations of radiation effects which are those of X-rays and γ-rays, the cell is chiefly imagined to be a fixed organization with uniform structure. Nothing is further from the truth. It is a complicated, dynamic system with many contributing factors which are necessary to its life, and these are in delicate balance or equilibrium. To speak glibly of absorption of quanta of energy by the cell is like saying another dynamic system, your automobile, won't run because it has absorbed some energy due to an injury. Its defect may occur in dozens of places and, as we know more about the mechanism of the automobile than we do of that more complicated, dynamic system, the cell, we can explain the course of the injury to it. We can then find a cracked spark-plug or some other defect, but as to the cell, we don't know enough about the mechanism to say what happens, and to say that energy quanta are absorbed only befogs the question by giving a name to a process which is not understood, and, if repeated enough, has the effect of obscuring speculation into the mechanism.

Certain generalization, however, may be made. Immature cells, and dividing cells are more easily affected by radiations in general. The effect upon the chromosomes is striking, but it may be only one of the ways in which biologic material responds to radiation. Muller himself warned that "not all the effects of radiation in killing organisms or disturbing their development are referable to changes either of the class of gene mutations or chromosome re-arrangements."

On highly differentiated cells the effect of injury from radiation is of such character as that resulting from other injuries or chemical poisons. When the blood supply is restricted or inhibited by radiation, it has even been suggested that radiation effects on a complex tissue are the results of the action on the circulation, but later studies disproved this. For the pathologist to reconstruct the mechanism of action of radiation from the dead, fixed tissue under the microscope is impossible, and the pathologic picture is in no fashion specific, for it can be produced by other injury or poisons.

The lymphocytes of the blood are highly susceptible to radiation, and if a very large neutron dose is given they disappear; with a smaller dose some of them disappear; if the dose is not great they recover their former numbers. Is this recovery due to repair processes on some injured cells or have enough of the mother cells escaped to make a nucleus of a new accretion? If they were a flock of ducks subjected to a blaze of gunshot, some may have been injured to recover later; some may have escaped the gunfire completely to form the nucleus of a new flock. The explanation escapes us: it may be one or both.

At various dose levels a change in behavior occurs in irradiated cells. At the highest dose level the result is quick death, presumably by severe breakdown in the physicochemical system of the protoplasmic cells; at lower levels, death results but a longer time is required for the upset of the machinery of the dynamic system of the cell. The changes may be transitory and reversible or they may be permanent and irreversible where the radiation injury disappears but leaves the tissue in a state of lowered resistance to further radiation. There is no single type of response which can be called the characteristic effect of radiation.

One of the interesting results is the production of cancer in rats by neutron radiation. In one series of 36 rats, 11 showed malignant tumors after continued radiation. The greater the number of days exposed the greater was the incidence of the cancer. In another series, 92 rats of 200 treated developed cancerous tumors. Rabbits and dogs did not have this susceptibility to the production of tumors, so no definite conclusions can be drawn in regard to the susceptibility of tumors in humans, but it is suggestive.

Much has recently been said about the great value that the atomic bomb discovery will be to the study of cancer, but does it really promise more than the discovery of other destructive forces? Its only present

promise is to popularize the use as "tracers" of radioactive material which we have been producing by the cyclotron for eight years at least. Radioactive sulfur and radioactive phosphorus, the ones most useful in biological study, are easy to produce: radio-carbon is more difficult. To use radioactive tracers merely to tell by Geiger counters into what organ the radioactive substance goes, gives little information; almost each experiment with radio-tracers requires a special chemical technique in addition to get real information.

Radio-phosphorus has been produced by the cyclotron and used in the treatment of leukemia and polycythemia for a number of years with considerable success. Its effects upon cancer have not been encouraging. Radio-iodine produced by the cyclotron has been also used in the treatment of certain forms of goiter with considerable effect. The atomic pile may produce these substances, but they are not new.

The real value of the new interest in radiation effects lies in the effort to explain their action in stopping the growth and division of young, immature cells. Cancer is a problem in cell division and cancer cells are relatively immature, rapidly-dividing cells. If their division can be inhibited, their relatively short life and weakness will cause them to die and, if no new cells are formed, the cancer will diminish and disappear. If the neutron radiation study shows the mechanism whereby lymphocytes are prevented from reproduction, the mode of prevention of cancer cell division is at hand. It may then be possible to apply and produce these conditions within the cancer cells, even without radiation, and so produce a stoppage of cell division and growth.—And that is our aim and hope.

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DEFORMATION IN RELATION TO TIME, PRESSURE AND TEMPERATURE.

BY

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Research work on the properties of materials within the scope of the above title has been reported in many hundreds of published papers. Engineering data and theory on the behavior of elastic and fluid bodies are sufficient for most practical problems. Yet to the physicist and to the mathematician the general problem is far from solved. The physicist is still unable to relate the properties of a body to those of its constituent molecules and atoms, the mathematician lacks a general function of which the known laws governing elasticity and hydrodynamics are special cases. Even the engineer feels the lack of information on plastic behavior in some problems. It is the purpose of this paper to outline a possible general framework into which the special laws of deformation may be fitted, paying particular attention to the little explored field of plastic behavior and not excluding the thermodynamics involved.

Reviews of much of the previous work in this field may be found in such texts as Bingham's "Fluidity and Plasticity" (1922), Houwink's "Elasticity, Plasticity and the Structure of Matter" (1937) and Nadai and Wahl's "Plasticity" (1931). In none of these digests of experimental data is there an attempt to formulate general physical laws. The absurd $D::(S - S_0)$ is still quoted although this assumes that the deformation is zero (really negative) until the stress S exceeds a lower limit S_0. Maxwell's "time of relaxation" is still used despite the fact that Maxwell himself questioned its validity. Even in current work, combinations of springs with leaky pistons are invoked to represent the behavior of plastics. It is general practice to lump all deformation rather than to separate it into the three stages which always exist. I know of no attempt to distinguish between adiabatic and isothermal deformation, in either theory or experiment. Nor are the powerful and exact energy relations of thermodynamics brought into play.

A few general deformation functions have been advanced. The writer in 1920 found that for his data on pitches, log deformation was a linear function of log time and of log force, that is $D = at^nF^m$ and that relaxation followed the same law.[1] Special values of n and m give the special laws for fluids and elastic solids. Later the function D — log

[1] *Jour. Franklin Institute*, 1921, p. 679 and 1943, p. 513.

$(1 + D) = atF$ was suggested[2] as an alternative not involving powers of the independent variables t and F. More recently Goranson[3] has deduced the hyperbolic sine function to represent a creep rate as the resultant of molecular displacement and fixation, in a stressed solid. I now find that the empirical logarithmic relation of 1920 may be easily deduced from the exact defining equations in common use. The addition of temperature to the independent variables permits a general survey of fundamentals. Some very precise data (including thermal) on steel tape are analyzed to illustrate the applications of these principles.

General Conditions. The body deformed is assumed uniform in composition, temperature and pressure throughout but not necessarily in a single phase, solid, liquid, gas or polymer. Uniformity of temperature means also that RT or any function of temperature and any constants (for example density and molecular weight) will be the same throughout the body. Even within the unit cell, the atoms and electrons are in equilibrium with the energy density of space at that temperature, which is $1.91 \times 10^{-15}\ T$ ergs/cc. This times the velocity of light, 3.0×10^{10} cm./sec., is the rate of impact or $5.73 \times 10^{-5}\ T$ ergs/cm.^{2}/sec. At 27° C., this amounts to about 0.67 cal./cm.^{2}/min., nearly a third that of direct sunlight. Bodies recovering in temperature, need draw only a fraction of the heat from surrounding objects.

Uniformity of pressure means not only freedom from residual strain but that at every point and in every direction the resultant of the four pressures is zero; the applied external pressure p is the independent variable, the cohesive pressure P holds a solid or liquid together despite the outward kinetic or thermal pressure κ and the contact or distending pressure π. $p + P = \kappa + \pi$. Internal latent heat is due to the cohesive pressure, the chief constituent of structural potential and of entropy. Kinetic pressure is thermal energy per unit volume or RT times concentration/molecular weight. Contact or distending pressure is the chief factor in compressibility but of course cohesive and kinetic pressures are also involved.

Since $p + P + \kappa + \pi = 0$, every derivative of that sum ($dp + dP + d\kappa + d\pi$ etc.) is also zero. So is every multiple of that expression or of its derivative whether the multiplier is a constant or a variable. The implications as regards the work, entropy and energy of deformation are of the greatest importance. Melting depends chiefly upon the relation between cohesive and kinetic pressures as does the so-called activity of a substance. The gas law $pv = RT$ simply states that $p = \kappa$ when P and π both become zero. Any equation of state may be regarded as a statement that the resultant of the four pressures is zero.

[2] *J. Wash. Acad. Sci.*, March 1928.

[3] "Flow in Stressed Solids," *Bull. Geol. Soc. Am.*, 1940, p. 1029.

Entropy and the second law are simply related to kinetic and cohesive pressures, as shown later in connection with the energy of deformation.

Three distinct stages must be recognized in every deformation although one or two of them may be relatively small in special problems. First there is the instantaneous yield immediately following the application of force, necessarily adiabatic since there is no time to acquire or disperse heat energy. Next there is the slower yield approaching the third and final stage of rest or of steady flow. All three stages follow the same general laws but with vastly different parameters. Much confusion has resulted from failure to treat these three stages separately. Although the transition from one stage to the next is usually too rapid or too slow for accurate observation, a long range of data in any one stage is sufficient to set its limits.

The time limit for the first stage is evidently set by the velocity of sound, the force limit is rupture, the temperature limit melting. "Bouncing putty" in plastics and the ricochetting of projectiles from a water surface are not unreasonable from this angle. The second stage is a period of readjustment and reorientation of internal forces, characterized by both elastic and fluid behavior. In the flow of fluids the first two stages are negligibly small in most cases but in some problems inertia may be an important factor. All three stages are reversible, each following the same law as the initial deformation usually with little hysteresis.

BASIC RELATIONS.

Since any deformation may be resolved into a compression (negative, expansion) and a shear and both follow the same general law with different constants, both may be represented by the same variable. For simplicity, the variable y (associated with yield) will be used to represent all deformations, whether changes in length, area, or volume or of shear angle. For a change in length l, $y = (l - l_0)/l_0$, the change in length per unit length, a dimensionless fraction in all cases.

Deformation is a function of either elapsed time t, pressure (or force) p or change in temperature $T - T_0$. It is customary to write deformation simply proportional to these, for example $l - l_0 = l_0\alpha(T - T_0)$ for thermal expansion—a strictly linear relation, while a graph of data always shows more or less curvature. A simple and very satisfactory way of representing this relation is to plot log y against log $(T - T_0)$ and find the slope of the resulting straight line; this is equivalent to putting $y = \alpha(T - T_0)^n$. The exponent n takes care of even large curvatures. With $n = 1$, the deformation reverts to the simple linear relation above. The coefficient α is the same as before, namely the deformation per degree of temperature variation. Its dimension is the inverse of the variable with which it is associated, but in the logarithmic relation it is a variable as experimental data show that it should be.

With this slight generalization to provide better accord with observed data, the defining equations for the coefficients become

$$\frac{dy}{y} = n\frac{dt}{t}, \qquad y = \gamma t^n, \qquad \log y = n \log t + \log \gamma,$$

$$\frac{dy}{y} = m\frac{dp}{p}, \qquad y = \beta p^m, \qquad \log y = m \log p + \log \beta, \tag{1}$$

$$\frac{dy}{y} = r\frac{dT}{T}, \qquad y = \alpha T^r, \qquad \log y = r \log T + \log \alpha.$$

Being defining equations, (1) are of course exact in differential form. In integral form they hold to well beyond first order precision over considerable ranges of the variables. Each parameter is a function of the other two independent variables, these being partial differentials.

These defining equations may be assembled as in (2)

$$\begin{aligned} y &= at^n p^m T^r, \\ \log y &= n \log t + m \log p + r \log T + \log a, \\ \frac{dy}{y} &= n\frac{dt}{t} + m\frac{dp}{p} + r\frac{dT}{T}. \end{aligned} \tag{2}$$

This is the form found by the writer (in t and p) in 1920 to well represent experimental data on plastic yield, and by many others in many diverse fields since. In view of the above direct derivation from equations of definition, close agreement is to be expected. The problem is not one of finding a function but of interpreting and evaluating parameters.

Energy relations concerning the work, heat and stored energy changes involved in deformation may be derived from the Gibbs expression for thermodynamic potential, $U - TS + pv$ (when there are no chemical changes) by differentiation. The derivative is seen to consist of two parts, each separately zero, corresponding to the first and second laws of thermodynamics. Thus

$$dU - TdS + pdv = 0, \qquad \text{by the First Law,} \tag{3}$$

$$-SdT + vdp = 0, \qquad \text{by the Second Law.} \tag{4}$$

In simple terms, (3) states that the sum of work done, heat generated and energy stored is always zero (conservation of energy). From (4), an additional relation between heat and work, may be directly derived a number of the fundamental relations of thermodynamics which have been amply verified by experiment. This simple derivation appears to have escaped previous writers. There are two general classes of problems, those with and those without change of phase.

With *two phases* present, say a liquid and its vapor, solid and liquid or either and its solution, T and p are the same in the two phases but

specific volume v and entropy S may differ widely. The ratio $dp:dT$ varies with the temperature. Writing (4) as $S_1 = V_1 dp:dT$ and $S_2 = V_2 dp:dT$ for the two phases and subtracting, $S_2 - S_1 = (V_2 - V_1)\, dp:dT$. But $T(S_2 - S_1) = L$, the latent heat of the change of phase, so that the general and exact relation between it and change of pressure with temperature is

$$L = T(S_2 - S_1) = T(v_2 - v_1)\frac{dp}{dT} = p(v_2 - v_1)\frac{d \log p}{d \log T}, \qquad (5)$$

applying to any two phase system having identical material in the two phases. (5) of course is only the familiar Clausius equation in form but I submit that the above derivation is far simpler than the usual one of following a work process around a Carnot cycle. When one specific volume is comparatively small, $pv_2 = RT$ and (5) becomes the Le Chatelier equation

$$\frac{d \log p}{d \log T} = \frac{L}{RT}. \qquad (6)$$

Other slight modifications give the reaction isochore and the Arrhenius-Boltzmann equation for atomic heats.

Deformation is concerned with cohesive pressure and hence with internal latent heat, $L - RT$. This, in energy per mole, is readily reduced to energy per unit volume and thence to force per unit area or pressure. In saturated solutions the damping effect of solvent on solute is taken account of by using activities in place of concentrations. Equation (4) amounts to two very simple statements: (*a*) $d\kappa = dp$ in either phase and (*b*) the entropy per unit mass is the same in both phases. This may be seen by converting temperature to kinetic pressure (κ) through the factor R/molecular weight $\times$ specific volume. Conversely, if (*a*) and (*b*) are axiomatic, they substantiate (4) and therefore (5).

Single phase thermodynamic changes by (4) are relations between pressure, temperature, volume and entropy. In many cases entropy may be expressed in terms of the other variables and (4) becomes simply an equation of state in differential (exact) form. Since $TdS = dQ$ by definition and dQ is readily evaluated in most practical problems, (4) is of wide application in deformation theory.

When a body is deformed, the work done, stored energy and heat lost or generated not only sum up to zero (3), but there are quantitative relations between these energy packets given by (4). When two phases are present, there are large and abrupt changes in specific volume, cohesive pressure and specific heat from one phase to the other, but within a single phase these variables vary continuously and we have not only $dp:dT$ but dS and dv. From (4) $dT:dp = dv:dS$ for such continuous small changes. But $TdS = dQ = C_p dT$ (with no change in state) and

by (1) $dv = \alpha v dT$, hence (4) gives directly the celebrated Thomson equation,

$$\frac{dT}{dp} = \frac{T\alpha v}{C_p}, \tag{7}$$

for the change in temperature caused by a change in pressure. This was derived by him about 1850 using many pages of differential equations, still followed in even modern texts. The above derivation is far simpler. This equation of state (7) has been amply verified by precise measurements in both tension and shear (torsion) but has been surprisingly little used. In (7), volume is the volume of unit mass (specific volume) since C_p is so limited.

It is to be noted that (4) holds equally for a change of state and for deformation within any one phase, whether that deformation be elastic or viscous. And change of phase includes polymerization and depolymerization as well as melting and vaporization. Since plastic deformation concerns both change of phase and elastic deformation, (4) is more useful in the form of dimensionless energy ratios (still perfectly general),

$$\frac{pdT}{Tdp} = \frac{d \log T}{d \log p} = \frac{pdv}{TdS} = \frac{dW}{dQ}. \tag{8}$$

To proceed further it is necessary to specify the work dW and the thermal energy dQ (which is either L or $C_p dT$) from observed data to obtain the complete solution of (3) and (4). The substitutions $pdv = p\beta v dp$, $Tdv = T\alpha v dT$ and $TdS = T\alpha S dT$ are useful in many problems. Care must be taken to distinguish adiabatic from isothermal coefficients. Isothermal $\beta = k\beta$ adiabatic, while for the α's this ratio is $1 - k$ where $k = C_p : Cv$, the ratio of specific heats. Equations (1) hold for either the isothermal or adiabatic condition. With most metals, k differs little from unity. With $k = 1.1$, a projectile on impact on armor would develop ten times as much heat energy as it had kinetic energy. In (1) and (2) it may be noted that the relations between p and v are consistent with the function pv^k = constant or $\log p + k \log v$ = constant, known to hold for metals as well as gases to within the uncertainties of observation.

A wire is cooled by stretching or by torsion, rubber is warmed—in strict conformity to (7), the chief variable being the thermal coefficient α. Beyond the elastic limit, where the deformation is no longer elastic and reversible, (4) and (7) still hold but the thermal energy dQ contains an additional factor, the heat of rupture or depolymerization, similar in nature to the latent heat of change of phase. In purely viscous flow, heat and work are equal while in elastic deformation the heating or cooling may be many times the work done. By precise calorimetric

methods, it should be possible to separate the elastic and viscous effects in such bodies as twisted bars of lead for example.

The selected data on steel tape given below will serve to illustrate the rather sketchily outlined general principles deduced above. These data were chosen because temperatures as well as yields were observed. In another experiment the yield and relaxation of the same tape was observed over a period of four months. As the writer can testify, the Washington monument tests on steel tape were most impressive and convincing. An initial yield of *two feet* would be followed by a continuously decreasing rate of yield, becoming slow enough to measure with a micrometer microscope only after half a minute had elapsed.

STEEL TAPE DATA.

The observed data used for the following table and curves were selected from many sets taken by Van Orstrand in 1907 on a tape hung in the Washington Monument and used by the writer in a paper on

TABLE I.

Thermodynamic Data on Steel Tape.

Load (kg.)	Yield (cm.)	Temp. Drop	Yield (cm./kg.)	dT/dF (deg./dyne)	α (linear) from (7)	$T\alpha$	ΔW	ΔH	ΔS
10	7.67	.085°	.7670	86.7×10^{-10}	12.09×10^{-6}	.00339	3.147	40.90	.156
20	15.37	.17	.7685	96.0×10^{-10}	13.39×10^{-6}	.00376	3.159	45.29	.161
30	23.09	.29	.7697	163.2×10^{-10}	22.76×10^{-6}	.00639	3.171	76.99	.274
40	30.85	.44	.7712	125.0×10^{-10}	17.43×10^{-6}	.00489	3.184	58.97	.210
50	38.63	.55	.7726	102.1×10^{-10}	14.24×10^{-6}	.00400	3.197	48.17	.172
60	46.45	.65	.7742	90.8×10^{-10}	12.66×10^{-6}	.00355	3.210	42.84	.153
70	54.31	.73	.7759	86.7×10^{-10}	12.09×10^{-6}	.00339	3.224	40.90	.146
80	62.20	.82	.7775	86.7×10^{-10}	12.09×10^{-6}	.00339	3.237	40.90	.146
90	70.12	.90	.7791	86.7×10^{-10}	12.09×10^{-6}	.00339	3.252	40.90	.146
100	78.08	.99	.7808	86.7×10^{-10}	12.09×10^{-6}	.00339	3.266	40.90	.146

Deformation and Temperature (*J. Wash. Acad.*, March 19, 1929). The dimensions of the tape were 14,685 × 0.640 × .0165 cm., volume 155.07 cc., section 0.01056 cm.^{2}, mass 1,194 grams, length/mass 12.30 cm./gm., density 7.70 g.m/cc. The temperature drop was taken immediately on application of the load, from an electrical resistance measurement. This and the specific heat, taken as 0.115 cal./gm./deg. throughout, are not appreciably affected by strain and only in the fourth decimal by the 0.1° change in temperature. Initial temperatures were all near 7.5° C. or 280.6° K. and readings were corrected to that base. Those readings of length taken four minutes after the load was applied are here used. The ultimate strength of this tape was between 150 and 200 kg. The observed data are reproduced on the first three columns. The fourth column shows the slight but uniform increase in yield. The data of the third column is shown plotted against that of the first in the figure. The fifth column was obtained by differentiating that curve.

Multiplying dT/dF by C_p/Tl_1, gives the sixth and by C_p/l_1, the seventh columns. ΔW is the mechanical work ($\frac{1}{2}p\Delta l$) and ΔH the thermal energy ($C_p dT$) lost per gram mass of tape per gram weight of tension. The entropy drop $\Delta S(=\Delta H/T)$ varies with the temperature drop while C_p does not. Not shown are $\beta(=dl/l$ per dyne$)$ = 53.3 to 54.3 $\times$ 10^{-12} and $p\beta$ varying from .000522 to .000532 as the load is varied from 10 to 100 kg. The ratios $dH:T\alpha$ and $dW:p\beta$ are of course constant and equal to each other throughout, since each is equal to $v_p dp = S_1 dT$.

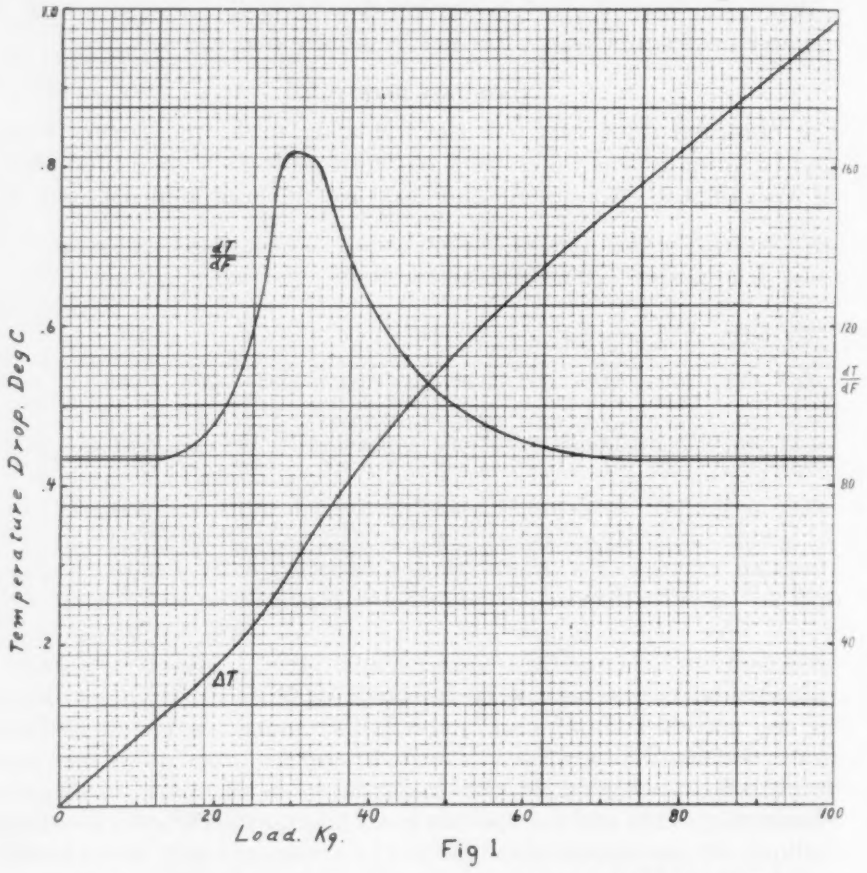

FIG. 1. Variation of temperature drop with load, and its derivative.

The graph clearly shows the abrupt large increase in $dT/dF(T\alpha v_1/C_p)$ at a load of about 30 kg., which is only about one-sixth of the breaking strength. Below 20 and from 70 to 100 kg. load the heat absorbed and expansivity have the same values which are but about half those at 30 kg. The transition is more abrupt on the side of lower loads. While α is varying by nearly 100 percent. and back, β is increasing steadily by

only 2 percent. The stretch caused by loading is about 530 times the change in length caused by the drop in temperature and is of opposite sign. The departure of even this steel tape from perfect elasticity (stress:strain ratio independent of stress and of time) is to be noted. Instead of $n = 0$ and $m = 1.0$ in the general formula, strain: $:t^nF^m$, the data give $m = 1.0077$, $n = 0.0016 (n = 0.0012)$ for the relaxation curve. The variation in yield per kg. (fourth column) might be removed by taking the lengths at smaller times for higher loads instead of taking them all after 4 minutes.

The heat absorbed by stretching (a damping of heat motions?) is from 13.0 to 24.3 times the energy used in the process. The ratio $T\alpha:p\beta$ of course varies in the same way between the same limits. The constant value of $SdT = vdp$, that is of $\Delta H/T\alpha = \Delta W/p\beta$, is 12,060 throughout. The work done is stored as potential energy and this is but slightly affected by the return of the tape to its original temperature. Its gradual increase, measured for four months, is too slow to be of consequence in physical or engineering problems. In other words, heat absorbed by mechanical work depends chiefly on the ease with which molecules may be *pushed* apart not *pulled* apart, the latter is not an important factor. Entropy varies inversely as this "push-ability," since $T\alpha S = C_p$, and T, v and C_p are sensibly constant over such short ranges of temperature.

Despite the large loss of thermal energy on stretching, none of it is potential heat for with recovery of temperature, only the mechanical potential energy remains. The large change in kinetic pressure does not affect the cohesive pressure.

The slow yield and relaxation of the same tape over a period of four months was further studied by Van Orstrand and H. D. Ayres by interference methods. In this work the original length was 27.906 cm., the temperature 27.1° C. and extensions were measured in wave lengths of He 5,875.63. Some of this data, taken in 1908, was published in *Bull. Geol. Soc. Am.*, 1923, p. 304. I have taken their data for 10 kg. load from 1.25 minutes to 126 days. The initial stretch of about 440 wave lengths was followed by a slow further yield of about 4.5 wave lengths, which 12 observations between the 101st and 126 days showed was still continuing at the rate of 0.0094 wave lengths per day. Following unloading, the relaxation curve was a duplicate in reverse of the load curve. These time effects are in line with the observed sag of old steel truss and suspension bridges.

Of chief interest here is the transition between the first and second stages of yield. Plotting log $(l - l_0)$ against log t (in minutes) with $l_0 = 439.00$ wave lengths, gives a line of slight curvature. The correction to l_0 to make the relation strictly linear is found by taking three equally spaced (log t) points on the curve. Then by simple algebra that correction is $(l_1l_3 - l_2^2)/(l_1 - 2l_2 + l_3) = 0.44$. With $l_0 = 439.44$,

the plot is linear from $t = 1.25$ minutes to 126 days with a slope of $n = 0.2397$. The formula $l - l_0 = at^n$ is sufficient down to 1.25 minutes at least and there is no evidence that it would not continue to hold below one minute to where adiabatic effects enter.

While this power-time function represents the second stage of deformation very well, it does not have the simple theoretical foundation that the force and temperature functions have. According to molecular physics, if rates of migration and fixation are reciprocal functions of identical arguments (force:energy), then their difference, the rate of deformation, is the hyperbolic sine function of that argument. However such reasoning is highly speculative and very precise observations on carefully selected materials would be required to distinguish between log and sinh function.

Other important problems, both experimental and theoretical, awaiting solution relate to the evaluation of all the parameters of (1) and (2). There seems to be little reliable data on even the temperature coefficients of Young's modulus and other physical properties of bodies. Many of these properties are known to be approximately linear functions of entropy but the theoretical foundation is lacking. Vapor pressure, viscosity, reaction constant, electrical resistance and many others are nearly linear in l/T, as are also d log function: d log T and therefore $dQ:dW$. The problem is to express each such property as a converter of energy, in terms of the ratio of heat to work at various temperatures. Such expressions are easily written, following (4).

SUMMARY.

The linear logarithmic relations between deformation and temperature and pressure, previously empirical, have been directly derived from the defining equations for compressibility and thermal expansivity and shown to apply to all three stages of every deformation.

Gibbs' thermodynamic potential function is shown to lead directly to simple and exact expressions for the energy of change of phase and of deformation within any one phase. The latter, a general equation of state, amounts to a simple relation between the four internal pressures.

The derived deformation and energy functions are applied to some experimental data on steel tape which included thermal and relaxation observations.

Thermodynamic relations governing both elastic and viscous behavior are developed. Plastic behavior involves both single phase and multiple phase applications of the second law. Precise thermal measurements should give the constants.

A SUMMARY OF DIMENSIONAL ANALYSIS.

BY

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Introduction. The idea that is emphasized in the following treatment is that dimensional analysis may be regarded as the mathematical theory of a class of functions that is characterized by a generalized type of homogeneity. It is felt that this point of view most clearly exhibits the scope of dimensional analysis, and that it avoids philosophical questions that have led to much controversy.

Dimensional Homogeneity. It is assumed, for the sake of generality, that there are n fundamental physical dimensions. If the corresponding symbols $[M]$, $[L]$, $[T]$, $[\Theta]$, $\cdots$, $[\Psi]$ are arranged according to a fixed convention, they may be known by their positions in the sequence, and the explicit use of the symbols consequently becomes superfluous. Thus, any dimensional expression $[M^{a_1}L^{a_2}T^{a_3}\Theta^{a_4}\cdots\Psi^{a_n}]$ is concisely represented by its exponents $(a_1, a_2, a_3, a_4, \cdots a_n)$, and, in conformity with common algebraic terminology, this set of numbers may be called a "vector." Accordingly, there is associated, with any scalar variable x, a constant "dimension vector" a_r that has n real components. If $a_r = 0$, the variable x is said to be "dimensionless."

The function of the dimension vector is to tell how the measure of a scalar magnitude changes when the basic units of measurement are subjected to prescribed changes. Abstractly conceived, a scalar entity is a one-dimensional space whose points are the possible magnitudes of the entity. By processes of physical measurement, it is possible to place the points of this space in $(1-1)$ correspondence with a set of real numbers; i.e. it is possible to establish a real coordinate system for the space. This coordinate system may be subjected to a special group of transformations which is derived by aribtrary changes of the basic units of measurement. To perceive the exact nature of these transformations, we let the units of measurement which correspond to the fundamental dimensions $[M]$, $[L]$, $[T]$, $[\Theta]$, $\cdots$, $[\Psi]$ be altered by the respective factors $\exp(-k_1)$, $\exp(-k_2)$, $\exp(-k_3)$, $\exp(-k_4)$, $\cdots$, $\exp(-k_n)$, where $k_1, k_2, k_3, k_4, \cdots, k_n$ denote the components of an arbitrary vector k_r, and the symbol $\exp(-k_r)$ denotes an arbitrary positive [1] base λ raised to the power $-k_r$. The coordinate x of a scalar magnitude whose dimension vector is a_r then undergoes a transforma-

[1] For numerical calculations, it is convenient to set $\lambda = 10$, but, for mathematical demonstrations, it is better to let λ be the base of natural logarithms.

tion $x \rightarrow \bar{x}$ which is defined by the following equation:

$$\bar{x} = x \exp \sum k_\nu A_\nu. \qquad \text{Eq. (1)}$$

Now x and $\bar{x}$ represent the same magnitude; the transformation $x \rightarrow \bar{x}$ merely results from a change of the units of measurement. Hence, if the form of an equation that relates scalar magnitudes is independent of the units of measurement, the equation must be invariant under all transformations $x \rightarrow \bar{x}$ which are defined by Eq. (1). More specifically, if a physical relationship is expressed by the equation[2] $y = f(x^1, x^2, \cdots, x^m)$, and if the dimension vectors for the variables $x^1, x^2, \cdots, x^m, y$ are respectively $a_\nu{}^1, a_\nu{}^2, \cdots, a_\nu{}^m, b_\nu$, the relation

$$y \exp \sum k_\nu b_\nu = f\{x^1 \exp \sum k_\nu a_\nu{}^1, \cdots, x^m \exp \sum k_\nu a_\nu{}^m\} \qquad \text{Eq. (2)}$$

must be an identity in the vector k_ν.

Eq. (2) may be regarded as a precise and general definition of dimensional homogeneity. To the author's knowledge, this form of the definition has not previously been explicitly stated. It is the object of the following discussion to show that the theorems of dimensional analysis are mathematical consequences of the identity, Eq. (2).

It is interesting to observe that Eq. (2) may be alternatively expressed in a differential form. This fact is suggested by Euler's theorem on homogeneous functions. Since the demonstration is essentially the same as that which is commonly presented in derivations of Euler's theorem, the result is stated without proof:

"A differentiable function $y = f(x^1, x^2, \cdots, x^m)$ is dimensionally homogeneous if, and only if, it satisfies the differential equations

$$yb_\nu = a_\nu{}^1 x^1 \partial y/\partial x^1 + a_\nu{}^2 x^2 \partial y/\partial x^2 + \cdots + a_\nu{}^m x^m \partial y/\partial x^m. \qquad \text{Eq. (3)}$$

In the class of differentiable functions, Eqs. (2) and (3) are perfectly equivalent.

Several elementary theorems follow in an obvious manner from Eqs. (2) or (3). For example, it may be seen that a sum of several variables is dimensionally homogeneous if, and only if, each term in the sum has the same dimension vector as the sum. Likewise, it is evident that a product of several variables is dimensionally homogeneous if, and only if, the dimension vector of the product is the sum of the dimension vectors of the factors.

Similitude. Eq. (2) expresses the condition of dimensional homogeneity in a form that suggests application to studies of similitude. For this equation shows that the dependent magnitude y is altered by the factor $\exp \sum k_\nu b_\nu$ when the independent magnitudes $x^1, x^2, \cdots, x^m$ are altered by the respective factors $\exp \sum a_\nu{}^1 k_\nu$, $\exp \sum a_\nu{}^2 k_\nu$, $\cdots$, $\exp \sum a_\nu{}^m k_\nu$. Thus, if the latter factors represent the ratios of magnitudes between a model and its prototype, Eq. (2) tells immediately the scale effect upon the dependent magnitude y. It is an elementary

[2] The superscripts are merely indices; they do not denote exponents.

algebraic problem to calculate coefficients k_ν which will effect prescribed reductions of the magnitudes x^i of a physical system, provided that such coefficients exist. Unfortunately, the ratios of magnitudes between a model and its prototype must usually be of a restricted class, in order to insure the existence of the coefficients k_ν.

Products. Eq. (3) may be employed to prove the following well-known theorem: "A product $\pi = (x^1)^{c^1}(x^2)^{c^2} \cdots (x^m)^{c^m}$, with the dimension vector b_ν, is dimensionally homogeneous if, and only if, the exponents c^i are a solution of the linear algebraic equations

$$\sum_{i=1}^{m} a_\nu{}^i c^i = b_\nu. \qquad \text{Eq. (4)}$$

Much importance attaches to products of this type with $b_\nu = 0$; i.e. dimensionless products. A set of variables $x^1, x^2, \cdots, x^m$ may furnish several dimensionless products $\pi_1, \pi_2, \cdots, \pi_p$ which have the property that no one of these products is identically a product of constant powers of the others. The products $\pi_1, \pi_2, \cdots, \pi_p$ are then said to be "independent." A necessary and sufficient condition for this is that the exponents $c_1{}^i, c_2{}^i, \cdots, c_p{}^i$ of the respective products be linearly independent.[3] If p is the maximum number of independent dimensionless products that can be formed from the variables $x^1, x^2, \cdots, x^m$, the products $\pi_1, \pi_2, \cdots, \pi_p$ are called a "complete set" of dimensionless products of the given variables.

It follows from Eq. (4) that the exponents of a dimensionless product are a solution of the homogeneous algebraic equations

$$\sum_{i=1}^{m} a_\nu{}^i c^i = 0. \qquad \text{Eq. (5)}$$

Bridgman [4] has demonstrated that *any fundamental system* [3] *of solutions of Eq.* (5) *furnishes exponents of a complete set of dimensionless products of the* x's. Although there is some arbitrariness in the determination of a fundamental system of solutions of Eq. (5), the results may be rendered unique by specifying that the matrix of the solutions shall have the following form:

$$\left\| \begin{array}{cccccccccc} 1 & 0 & 0 & \cdots & 0 & 0 & c_1{}^{p+1} & c_1{}^{p+2} & \cdots & c_1{}^m \\ 0 & 1 & 0 & \cdots & 0 & 0 & c_2{}^{p+1} & c_2{}^{p+2} & \cdots & c_2{}^m \\ 0 & 0 & & & & & & & & \\ \vdots & \vdots & & & & & \vdots & \vdots & & \vdots \\ 0 & 0 & & & & & & & & \\ 0 & 0 & \cdots & \cdots & 0 & 1 & c_p{}^{p+1} & c_p{}^{p+2} & \cdots & c_p{}^m \end{array} \right\| \qquad \text{Eq. (6)}$$

[3] See "Introduction to Higher Algebra," M. Bôcher, Macmillan, 1938.

[4] "Dimensional Analysis," P. W. Bridgman, Yale Press, New Haven, 1931, p. 43.

Such a matrix always exists of the x's are arranged so that the matrix $(a_\nu^{p+1}, a_\nu^{p+2}, \cdots, a_\nu^m)$ has the same rank [3] as the m-column matrix (a_ν^i).

Each row in the matrix, Eq. (6), is a set of exponents in a dimensionless product of the x's. The complete set of dimensionless products that is determined by this matrix possesses the important practical feature that each of the variables $x^1, x^2, \cdots, x^p$ occurs in only one π-product. Buckingham [5] has pointed out that this property is advantageous for the experimentalist, since it permits him to regulate specific π-products while he keeps the others constant. This facilitates the plotting of dimensionless data.

A noteworthy corollary to the preceding theorem is the following:

"The number p of products in a complete set of dimensionless products of the x's is $p = m - r$, in which r is the rank [3] of the matrix (a_ν^i)." This fact was recognized by Bridgman,[4] but the belief still persists among many engineers and physicists that p is necessarily equal to $m - n$, in which n is the number of basic physical dimensions involved in the problem. Van Driest [6] has recently stated the rule in another way:

"The number p is equal to m minus the maximum number of x's that will not form a dimensionless product."

It may be shown that this is equivalent to the equation, $p = m - r$.

A theorem of some practical significance which is closely allied to the preceding theorems, and which is a result of the algebraic theory of Eq. (4), is the following:

"A π-product with a prescribed non-zero dimension vector b_ν exists if, and only if, the matrix (a_ν^i) has the same rank r as the augmented matrix (a_ν^i, b_ν). Then there are exactly $m - r + 1$ independent π-products with the dimension vector b_ν. The exponents of these products are linearly independent solutions of Eq. (4)."

Buckingham's Theorem. The great importance of the theory of products stems from Buckingham's theorem: [5]

"A necessary and sufficient condition that an equation $y = f(x^1, x^2, \cdots, x^m)$ be dimensionally homogeneous is that it be reducible to the form

$$Q = \phi(\pi_1, \pi_2, \cdots, \pi_p),$$

in which the π's are a complete set of dimensionless products of the x's, and Q is a dimensionless product of the variables $x^1, x^2, \cdots, x^m, y$.

As a lemma to this theorem, we state that, if $y = f(x^1, x^2, \cdots, x^m)$ is a dimensionally homogeneous function with a non-zero dimension vector b_ν, there exists a product of powers of the x's which has the same

[5] "On Physically-Similar Systems; Illustrations of the Use of Dimensional Equations," E. Buckingham, *Physical Review*, vol. IV, no. 4, p. 345, 1914.

[6] "On Dimensional Analysis and the Presentation of Data in Fluid Flow Problems," E. R. Van Driest, *Jour. App. Mech.*, vol. 13, no. 1, p. A-34, Mar., 1946.

dimension vector as y. This is an obvious consequence of Eqs. (3) and (4). The result can be proved directly from Eq. (2), without the hypothesis of differentiability, but the demonstration then becomes somewhat involved.

Buckingham's theorem expresses the existence of a correspondence between two point sets which are the ranges of the variables concerned. In discussions of existence theorems of this type, geometrical terminology facilitates the concise expression of ideas. Consequently, the variables $x^1, x^2, \cdots, x^m$ are regarded as coordinates in a space S. The equations,

$$x^i = x^i{}_0 \exp \sum k_r a_r{}^i, \qquad \text{Eq. (7)}$$

define a point transformation in this space; namely, a transformation which carries the point $x_0{}^i$ to the point x^i. The set of all points which can be derived from a point $x_0{}^i$ by such transformations will be called the k-space that is generated by the point $x_0{}^i$. It is evident that a k-space is generated by any one of its points, and that the space S is completely partitioned into non-intersecting k-spaces.

A complete set of dimensionless products $\pi_1, \pi_2, \cdots, \pi_p$ of the variables $x^1, x^2, \cdots, x^m$ may be interpreted to be coordinates in a space T. The relations between the x's and the π's define a map of the space S onto the space T. Buckingham's theorem is derived from the following algebraic theorem, which is easily demonstrated:

"There is a $(1-1)$ correspondence between the points of the space T and the k-spaces of the space S."

To proceed from this result to Buckingham's theorem, we note that any dimensionally homogeneous point function $y = f(x^1, x^2, \cdots, x^m)$ in the space S may be expressed in the form $Q = g(x^1, x^2, \cdots, x^m)$, in which Q is dimensionless. This is an immediate consequence of the foregoing lemma which asserts that there exists a product of powers of the x's with the same dimension vector as y. Now, by the definition of dimensional homogeneity, Eq. (2), a dimensionless point function Q in the space S is dimensionally homogeneous if, and only if, it is constant throughout each k-space. By the preceding theorem, this is equivalent to the condition that Q is a point function in the space T of a complete set of dimensionless products of the x's. Thus, it is demonstrated that the conditions expressed in Buckingham's theorem are necessary. The sufficiency of the theorem is at once obvious.

Buckingham's theorem practically closes the subject of dimensional analysis, for it effectively eliminates dimensional considerations by reducing physical problems to dimensionless forms.

Air Springs Proposed for Railroad Cars.—(*Compressed Air*, Vol. 51, No. 10.) Air springs that resemble automobile tires laid on their sides are being tested for railroad-car service by their originator, The Firestone Tire & Rubber Company, in conjunction with the Pullman-Standard Car Manufaturing Company. Following three private tests, they were given their first public tryout recently in a special run over the Chicago, South Shore & South Bend tracks between Michigan City and Chicago. On this occasion, one end of a Pullman-Standard laboratory car was equipped with the air springs and the other end with steel-coil springs. Precision instruments capable of checking the slightest variations in lateral sway and vertical motion caused by track irregularities made recordings at speeds ranging from 35 to 80 miles an hour.

The air spring has already been used on airplane landing gear, heavy truck trailers, passenger buses, and Bofors anti-aircraft gun carriers, and has been tested on numerous passenger automobiles. In service, the tirelike bellows are inflated and connected with an air reservoir. When the spring is depressed by reason of a wheel passing over a track irregularity, air is forced from it into the reservoir. The rebound is then snubbed by a control valve between the bellows and the reservoir, thus minimizing the shock to the passengers.

Conforming with normal practice as regards steel springs, the truck of the laboratory car is fitted with two sets of air springs. The journal-box springs—two air springs—receive the first impact from the rails. They are at the end of each axle and are 9 inches in diameter and 4 inches high, respectively. Each pair is connected with a common air reservoir. Beneath each end of the truck bolster—the principal lateral member of the truck—is another air spring, 20 inches in diameter and 4 inches high. All are inflated from an outside compressor in the same manner as an automobile tire. The pressure varies anywhere from 75 to 100 pounds, depending upon the load to be carried.

The springs are expected to improve passenger comfort, to outlast the coil spring, and to require less maintenance. In a joint statement, Roy F. Johnson and Roy W. Brown, research engineers for Pullman-Standard and Firestone, respectively, said: "This is the fourth trial run made with air springs under a railway passenger car. The first was made at speeds not higher than 35 miles an hour, to prove out in a preliminary way the safety of the device. The second run was made at speeds up to 55 miles an hour to check on various details of the application, and on the suitability of the recording instruments. The third run was made at speeds as high as 80 miles an hour, and that top was again reached on the fourth run.

"It should be understood that the air spring is still in early stages of experimentation. We are seeking a comparison of its fundamental advantages over the steel coil springs now in ordinary use. This will require many test runs over a period of several months and a variety of adjustments and corrections."

R. H. O.

LIGHTING DESIGN BY THE INTERFLECTION METHOD.

BY

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Proposed international terminology [1] used in the paper is as follows:
pharos F = luminous flux (lumen),
pharosage D = luminous flux density (lumen m^{-2}),
helios H = generalized brightness (pharosage per unit solid angle, expressed in blondels).

1. INTRODUCTION.

In the conventional method of lighting design, attention has been directed principally to obtaining a sufficiently high pharosage on a horizontal surface at table level. Within recent years, however, it has become increasingly evident that satisfactory lighting requires consideration of the helios distribution throughout the visual field. Thus modern lighting design must include the predetermination of helios of walls, floor, ceiling, and other surfaces. Until recently there has been no feasible method of calculating these quantities.

A previous paper,[2] however, made use of integral equations in determining the effect of multiple reflections among the various surfaces in the room. Tables were computed to allow the effortless determination of helios distributions. Thus it became possible for the first time to design lighting systems for best visual conditions without resorting to experimentation with full-size rooms or scale models.

These tables applied to three types of lighting:

(I) Wall lighting,
(II) Ceiling lighting,
(III) Floor lighting.

In (I), all the light from the luminaires reaches the walls; none hits the ceiling and floor. Similarly, in (II) and (III), the direct lighting from

[1] Parry Moon and D. E. Spencer, "Internationality in the Names of Scientific Concepts," *Am. J. Phys.* (1946); "An Unofficial Guide to Photometric Nomenclature," to be published; "A Study of Photometric Nomenclature," *J. Opt. Soc. Am.* (1946); "Photometric Nomenclature for the Post-war World," *Illum. Eng.* (1946). "International Names in Colorimetry," *J. Opt. Soc. Am.*, **36** (1946).

[2] Parry Moon and D. E. Spencer, "Light Distributions in Rooms," J. FRANKLIN INSTITUTE, 242, 111 (1946).

the luminaires is confined to a specified surface. These three sets of tables are basic. They determine the pharosage and the various helios values for three important types of lighting. Furthermore, the principle of superposition allows them to be applied to any distribution from the luminaires.

To extend the practical usefulness of the previous tables, we expanded them into 28 tables by the superposition method. The new tables are given in the present paper and constitute a convenient tool in the design of lighting systems employing most types of luminaires. If still other cases are needed, they can be obtained by superposition from the original tables, as explained in the following pages.

2. DEFINITIONS.

Customary treatments of lighting design consider the illumination of the "working plane" and divide lighting installations into "direct," "indirect," and "semi-indirect." Since none of these terms is defined with any exactitude and since this classification does not correlate with interflection theory, we find it necessary to formulate new definitions.

The integral-equation treatment allows the calculation of average pharosage on any surface. In the interests of simplicity, however, it is desirable to standardize on one particular surface and to make all ordinary calculations on the basis of this surface:

The principal surface is defined as the upper side of an imaginary horizontal plane at height $h/4$ above the floor, where h is the ceiling height and where the surface is limited by the walls of the room.

The average pharosage D_{AV} is equal to the total pharos F_W incident on the principal surface, divided by the area of the surface:

$$D_{AV} = F_W/S.$$

We define three fundamental types of lighting: Type I, Type II, and Type III:

Type I, Wall Lighting. Wall lighting is defined as lighting from perfectly diffusing wall sources.

There are two sub-classes:

Type Ia. Self-luminous walls. The walls, or parts of them, are made of translucent material lighted from behind.

Type Ib. Secondary sources. The walls reflect light that is incident on them from luminaires within the room. All of the pharos F_D from the luminaires reaches the walls and none reaches the ceiling or floor directly.

Type II, Ceiling Lighting. Ceiling lighting is defined as lighting from a perfectly diffusing ceiling.

There are two sub-classes:

Type IIa. Self-luminous ceiling. The ceiling, or part of it, is made of translucent material lighted from above.

Type IIb. Secondary source. The ceiling reflects light that is incident on it from luminaires within the room. All of the pharos F_D from the luminaires reaches the ceiling; none is directly incident on walls or floor.

Type III, Floor lighting. Floor lighting is defined as lighting from a perfectly diffusing floor.

There are two sub-classes:

Type IIIa. Self-luminous floor. The floor, or part of it, is made of translucent material and lighted from below.

Type IIIb. Secondary source. The floor reflects light that is incident on it from luminaires within the room. All the pharos F_D from the luminaires reaches the floor, none is incident on walls or ceiling. The light reaching the principal surface comes directly from the luminaires and from the upper walls and ceiling.

Note particularly the distinction between Types IIa and IIIb. Both may refer to lighting from sources built into the ceiling. But in IIa the ceiling panels are perfectly diffusing and therefore send considerable light to the walls, while in IIIb the luminaires are adjusted to send no direct light to the walls. Most lighting systems today do not fall in any one of these classes but are combinations.

3. THREE FUNDAMENTAL TYPES OF LIGHTING.

The integral-equation method leads naturally to expressions in terms of the three basic types of lighting: ceiling lighting, wall lighting, and floor lighting. In all cases, the average pharosage is

$$D_{AV} = fg\frac{F_L}{S}, \tag{1}$$

where D_{AV} = average pharosage (lumen m^{-2}) incident on the principal surface,
F_L = total pharos (lumen) from all the lamps,
S = total floor area (m^2),
S_1 = total wall area (m^2),
$f = F_W/F_D$ = interflectance (numeric) of the room,
$g = F_D/F_L$ = logance (numeric) of the luminaires,
F_W = total pharos (lumen) reaching the principal surface,
F_D = total pharos (lumen) from the luminaires.

The product of f and g is often called the *coefficient of utilization:*

$$k_u = f \cdot g. \tag{2}$$

The shape of the room is specified by its *domance* k_r:

$$k_r = \frac{h(l + w)}{2lw} = \frac{S_1}{4S}. \tag{3}$$

The solution of the integral equation for interflections gives D_{AV} and the various H's as functions of domance and in terms of H_{01}, H_{02}, or H_{03}, the helios values without interflections. Thus the tables contained in the preceding paper [2] list the following quantities:

H_1/H_{01}, H_2/H_{01}, H_3/H_{01}, D_{AV}/H_{01} (wall lighting);
H_1/H_{02}, H_2/H_{02}, H_3/H_{02}, D_{AV}/H_{02} (ceiling lighting);
H_1/H_{03}, H_2/H_{03}, H_3/H_{03}, D_{AV}/H_{03} (floor lighting).

Subscripts 1, 2, and 3 refer to walls, ceiling, and floor, respectively. Values of H_{01}, H_{02}, H_{03} are obtained from the known luminous output of the luminaires.

The previous tables may be employed directly in lighting design, but there seems to be a slight practical advantage in using the conventional formula, Eq. (1). The tables of D_{AV}/H_{01}, D_{AV}/H_{02} and D_{AV}/H_{03} are changed to tables of H/D_{AV}. This transformation is made in the present paper, to obtain a closer correlation between the new interflection method and the lumen method of design[3] that has been in use by practicing engineers. To employ the new tables in designing a lighting system, one decides on a value of D_{AV}, calculates f, and solves Eq. (1) for the total lumens F_L required from the lamps. All values of helios are then obtained by multiplying tabulated values of H/D_{AV} by the known D_{AV}.

If F_L in Eq. (1) is taken as the initial pharos and f and g are taken for the new, clean installation, then D_{AV} will be the initial pharosage immediately after the lighting system is put into service. The equation, however, is equally applicable to conditions at any later date. All three quantities (F_L, g, and f) depreciate; F_L decreases with time because of lamp blackening, g decreases because of the collection of dust on the luminaire and lamp, and f decreases because of depreciation of the paint and collection of dirt on the room surfaces. An analysis of these effects is given in another paper.[4]

(*I*) *Wall Lighting*

The first type of lighting to be considered is *wall lighting*. Here the wall is the light source, either because it is translucent, with lamps behind it, or because it reflects light thrown on it from some form of luminaires.

Let F_D = total pharos (lumen) from luminaires,
F_1, F_2, F_3 = pharos from luminaires to walls, ceiling, and floor, respectively (for a self-luminous panel, the panel is taken as the luminaire),

[3] W. Harrison and E. A. Anderson, "Illumination Efficiencies as Determined in an Experimental Room," *I.E.S. Trans.*, **11**, 67 (1916); "Coefficients of Utilization," *I.E.S. Trans.*, **15**, 97 (1920).

[4] Parry Moon and D. E. Spencer, "Maintenance Factors," *Illum. Eng.* **41**, 211 (1946).

F_W = total pharos incident on principal surface,
S = floor area (m^2),
S_1 = wall area (m^2),
S_p = panel area (m^2),
$(D_{AV})_1$, $(D_{AV})_2$, $(D_{AV})_3$ = average pharosage produced by F_1, F_2, F_3, respectively,
f_1, f_2, f_3 = interflectances for wall lighting, ceiling lighting, and floor lighting, respectively,
ρ_1, ρ_2, ρ_3 = average reflectances of walls, ceiling, and floor,
ρ_p = reflectance of luminous panel,
ρ_c = reflectance of ceiling between panels,
ρ_w = reflectance of wall between panels.

(*Ia*) *Self-luminous walls* made of diffusing glass or plastic. Without interflections,

$$H_{01} = F_1/S_1. \tag{4}$$

Also,

$$f_1 = \frac{1}{4k_r}\left(\frac{D_{AV}}{H_{01}}\right), \tag{5}$$

and for any helios H_i,

$$\frac{H_i}{D_{AV}} = \frac{1}{(D_{AV}/H_{01})}\left(\frac{H_i}{H_{01}}\right). \tag{6}$$

(*Ia*) *Panels in Walls.* Average reflectance of walls is

$$\rho_1 = \rho_p S_p/S_1 + \rho_w(S_1 - S_p)/S_1. \tag{7}$$

Imagine the actual arrangement of panels to be replaced by uniform luminous walls having reflectance ρ_1 and helios H_1. Calculations are made as with completely luminous walls. Finally, the true values of helios are found from the relations,

$$H_w = \frac{\rho_w}{\rho_1}(H_1 - F_1/S_1), \tag{8}$$

$$H_p = \frac{F_1}{S_p} + \frac{\rho_p}{\rho_1}(H_1 - F_1/S_1). \tag{9}$$

(*Ib*) *Reflecting walls* lighted from luminaires within the room.

$$H_{01} = \rho_1 F_1/S_1, \tag{10}$$

$$f_1 = \frac{\rho_1}{4k_r}\left(\frac{D_{AV}}{H_{01}}\right), \tag{11}$$

$$\frac{H_i}{D_{AV}} = \frac{1}{(D_{AV}/H_{01})}\left(\frac{H_i}{H_{01}}\right). \tag{12}$$

Type II, Ceiling Lighting

For Type IIa lighting with a complete luminous ceiling, the helios of the ceiling without interflections is

$$H_{02} = F_2/S. \tag{13}$$

By the definition of interflectance,

$$f_2 = \frac{(F_w)_2}{F_2} = \frac{(D_{AV})_2 S}{F_2}$$

and by use of Eq. (13),

$$f_2 = \left(\frac{D_{AV}}{H_{02}}\right). \tag{14}$$

The interflectance f_2 is identical with the previously tabulated ratio (D_{AV}/H_{02}). Also, any helios $H_i (i = 1, 2, 3)$ is obtained from

$$\frac{H_i}{D_{AV}} = \left(\frac{H_i}{H_{02}}\right)\frac{1}{(D_{AV}/H_{02})}. \tag{15}$$

Thus new tables of (H_i/D_{AV}) are obtained by dividing the previous tabulated values of (H_i/H_{02}) by (D_{AV}/H_{02}).

For IIa lighting from luminous panels, the average reflectance of the ceiling is

$$\rho_2 = \rho_p S_p/S + \rho_c(S - S_p)/S. \tag{16}$$

The actual installation is assumed to be replaced by a uniform luminous ceiling having the reflectance ρ_2 and the helios H_{02}:

$$H_{02} = F_2/S = gF_2/S. \tag{17}$$

If the panels are reasonably well distributed over the ceiling, the pharosage D_{AV} will be almost exactly the same with the fictitious ceiling as with the actual panels.[5] The entire procedure is therefore as with the complete ceiling except that the true helios of the ceiling between panels is

$$H_c = \frac{\rho_c}{\rho_2}(H_2 - F_2/S) \tag{18}$$

and the true helios of the panels is

$$H_p = \frac{F_2}{S_p} + \frac{\rho_p}{\rho_2}(H_2 - F_2/S). \tag{19}$$

For Type IIb lighting,

$$H_{02} = \rho_2 F_2/S, \tag{20}$$

[5] D. E. Spencer, "Exact and Approximate Formulae for Illumination from Troffers," *Illum. Eng.*, **32**, 596 (1942); "Illumination from Arrays of Rectangular Sources," *J. Opt. Soc. Am.*, **32**, 539 (1942); Parry Moon, "New Methods of Calculating Illumination," *J. Opt. Soc. Am.*, **33**, 115 (1943).

$$f_2 = \rho_2 \left(\frac{D_{AV}}{H_{02}} \right), \tag{21}$$

$$\frac{H_i}{D_{AV}} = \left(\frac{H_i}{H_{02}} \right) \frac{1}{(D_{AV}/H_{02})}. \tag{22}$$

Type III, Floor Lighting

(IIIa) For a luminous floor,

$$H_{03} = F_3/S, \tag{23}$$

$$f_3 = \left(\frac{D_{AV}}{H_{03}} \right), \tag{24}$$

$$\frac{H_i}{D_{AV}} = \left(\frac{H_i}{H_{03}} \right) \frac{1}{(D_{AV}/H_{03})}. \tag{25}$$

(IIIb) For lighting from luminaires that throw all their light to the floor,

$$H_{03} = \rho_3 F_3/S, \tag{26}$$

$$f_3 = 1 + \rho_3 \left(\frac{D_{AV}}{H_{03}} \right), \tag{27}$$

$$\frac{H_i}{D_{AV}} = \frac{\rho_3}{f_3 - 1} \left(\frac{H_i}{H_{03}} \right) = \left(\frac{H_i}{H_{03}} \right) \frac{1}{\left(\frac{D_{AV}}{H_{03}} \right) + \frac{1}{\rho_3}}. \tag{28}$$

The foregoing equations allow one to change from the previous tabulated values [2] to the more conventional engineering representation in terms of f and D_{AV}.

4. THE GENERAL CASE.

The foregoing treatment has been restricted to the three special cases of wall lighting, ceiling lighting, and floor lighting. The principle of superposition will now be applied to give the general case of an arbitrary pharos distribution from the luminaires. The total pharos (lumens) from the luminaires is denoted by F_D. This pharos is divided into three parts, F_1, F_2, and F_3, which reach the walls, ceiling, and floor respectively:

$$F_D = F_1 + F_2 + F_3. \tag{29}$$

The general case of lighting from any type luminaire is obtained from the three special cases by making use of the pharos ratios: (F_1/F_D), (F_2/F_D), (F_3/F_D). Then the interflectance to be substituted into Eq. (1) is

$$f = f_1 \left(\frac{F_1}{F_D} \right) + f_2 \left(\frac{F_2}{F_D} \right) + f_3 \left(\frac{F_3}{F_D} \right). \tag{30}$$

Any helios H_i may be written, by superposition,

$$H_i = \left(\frac{H_i}{H_{01}}\right) H_{01} + \left(\frac{H_i}{H_{02}}\right) H_{02} + \left(\frac{H_i}{H_{03}}\right) H_{03}. \tag{31}$$

Or it may be written

$$H_i = \left(\frac{H_i}{D_{AV}}\right)_1 (D_{AV})_1 + \left(\frac{H_i}{D_{AV}}\right)_2 (D_{AV})_2 + \left(\frac{H_i}{D_{AV}}\right)_3 (D_{AV})_3.$$

But

$$f = D_{AV}S/F_D,$$

$$f_i = (D_{AV})_i S/F_i,$$

and

$$\frac{(D_{AV})_i}{D_{AV}} = \frac{f_i F_i}{f F_D}.$$

Thus, for the general case,

$$\frac{H_i}{D_{AV}} = \frac{1}{f}\left[f_1\left(\frac{H_i}{D_{AV}}\right)_1\left(\frac{F_1}{F_D}\right) + f_2\left(\frac{H_i}{D_{AV}}\right)\left(\frac{F_2}{F_D}\right) + f_3\left(\frac{H_i}{D_{AV}}\right)_3\left(\frac{F_3}{F_D}\right)\right]. \tag{32}$$

5. PHAROS RATIOS.

In applying the general method of Section 4, one must know the three pharos ratios (F_1/F_D), (F_2/F_D), and (F_3/F_D) which specify what fraction of the direct light from the luminaires reaches the walls, ceiling, and floor. With the logically designed luminaires of the future, there is no reason why these ratios should not be fixed by the manufacturer at definite values such as 1, $\frac{1}{2}$, or 0. At present, however, luminaires are not designed. They are slapped together in a haphazard manner, and the light from them is not confined to any definite region nor does it follow any predetermined distribution. Under the circumstances, the calculation of the foregoing pharos ratios becomes a problem.

From test data on the luminaire, one obtains the pharosage distribution on the ceiling and on the floor for a single luminaire. Two isolux diagrams are plotted, one showing curves of equal pharosage (lumen $m.^{-2}$) incident on the ceiling from a luminaire, the other showing similar curves for the floor. If these plots are made on tracing paper, they can be placed on top of a plan of the room (Fig. 1). For each location of a luminaire, the tracing paper is moved to the proper position and the pharos to the ceiling (or floor) obtained by multiplying areas by pharosage values. In this way, the total pharos F_2 to the ceiling from all luminaires is obtained, also the total pharos F_3 to the floor. Subtracting the sum from the known pharos F_D from the luminaire, one obtains the pharos F_1 to the walls.

The foregoing method applies in all cases. Usually, symmetry allows considerable simplification; and in some cases, analytic methods can be employed. With spherical globes of uniform helios, for instance,

the pharos to a rectangular portion of ceiling, a corner of which is directly above the luminaire, is

$$\frac{F}{F_D} = \frac{1}{4\pi}\left[\tan^{-1}\frac{b\sqrt{a^2+b^2+c^2}}{ac} + \tan^{-1}\frac{c\sqrt{a^2+b^2+c^2}}{ab} - \pi/2\right], \quad (33)$$

where b and c are the dimensions of the rectangular portion of ceiling, and a is the distance from center of luminaire to ceiling. The same equation is used for the floor.

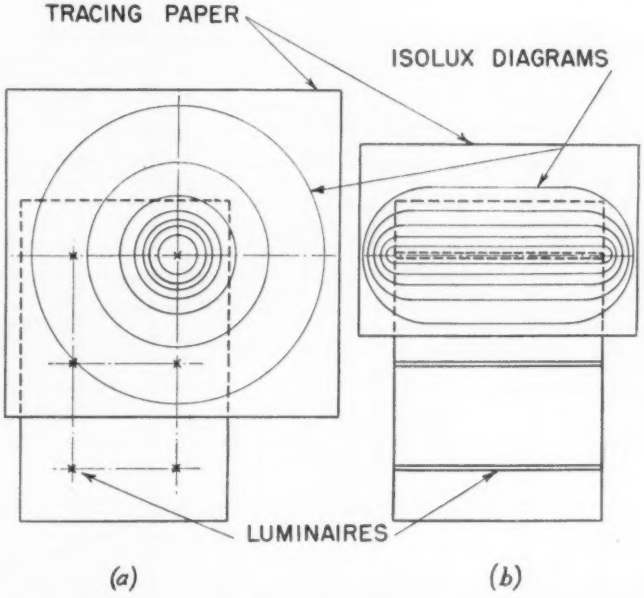

FIG. 1. Isolux diagrams superposed on floor plans of a room to allow the calculation of F_2/F_D and F_3/F_D. Areas are measured with a planimeter and total pharos to ceiling and floor is calculated. (a) Luminaires with incandescent lamps. (b) Continuous rows of fluorescent lamps.

With square rooms employing uniform diffusing globes with spacing equal approximately to the ceiling height h and with luminaires hung at a distance $a = h/4$ below the ceiling, we obtained curves of (F_2/F_D), and (F_3/F_D) as functions of domance k_r. These curves could be expressed by the empirical equations,

$$\frac{F_2}{F_D} = \tfrac{1}{2}(2k_r + 1)^{-\frac{1}{2}}$$

$$\frac{F_3}{F_D} = \tfrac{1}{2}(2k_r + 1)^{-\frac{1}{2}} \quad (34)$$

$$F_1/F_D = 1 - \frac{F_2}{F_D} - \frac{F_3}{F_D}.$$

The pharos ratios are plotted in Fig. 2 and were used in obtaining the tables for diffusing globe lighting.

6. CEILING LIGHTING, TYPE IIb.

New tables will now be given to provide maximum simplicity in lighting design. First consider ceiling lighting. The ceiling is assumed to be uniformly lighted by means of luminaires that send all their luminous output to the ceiling with no spill light on the walls.

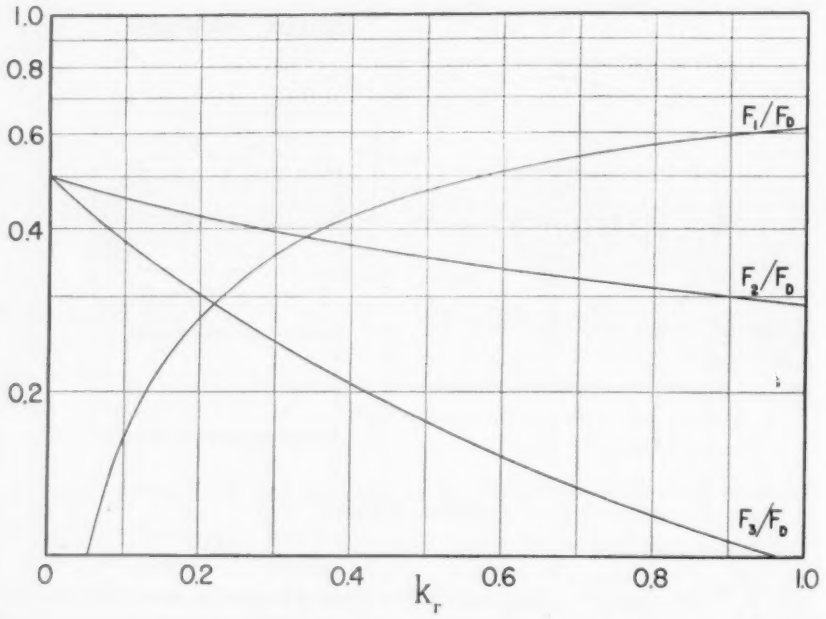

FIG. 2. Pharos ratios for uniform diffusing globes. F_1/F_D is the fractional part of the luminous output that is directly incident on the walls. F_2/F_D is the part that reaches the ceiling, and F_3/F_D is the part that reaches the floor.

The equations of Section 3, then apply, resulting in Tables I to V, inclusive. Table I gives the interflectance f; Table II, (H_{IT}/D_{AV}); Table III, (H_{IM}/D_{AV}); Table IV, (H_2/D_{AV}); Table V, (H_3/D_{AV}). The tables are given for two floor reflectances ($\rho_3 = 0.10$ and 0.30) and for a number of ceiling and wall reflectances. Values are listed for the domances 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1.0, 2.0, which covers the room shapes ordinarily encountered. Linear interpolation may be employed to obtain approximate values at intermediate domance and reflectances, or the tabulated results may be plotted as in Fig. 3. If log paper is used, the curves are all approximately straight lines.

7. PANEL LIGHTING.

Lighting is accomplished by means of perfectly diffusing panels built into the ceiling. The equations of Section 3 apply. Instead of using Eqs. (18) and (19) to calculate H_p and H_c from H_2, however, the engineer may find it convenient to have tables for these values of helios. Examples are given in Tables VI and VII for the particular case of $S_p/S = \frac{1}{5}$. This is a fairly common value for fluorescent lighting with built-in lighting elements running completely across the ceiling.

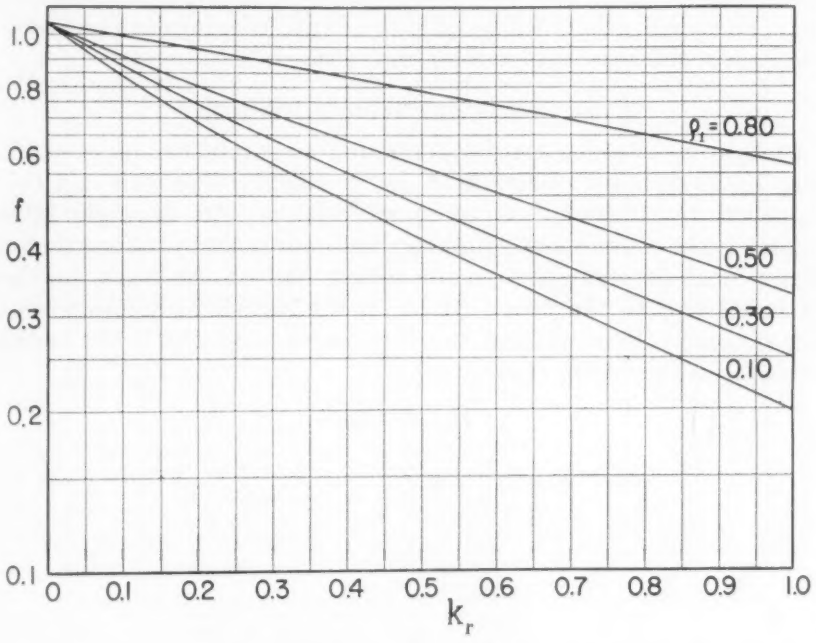

FIG. 3. Typical family of curves for interflectance. Ceiling lighting, Type IIb. Ceiling reflectance is 0.80, floor reflectance is 0.30.

If the width of the diffusing glass is $\frac{1}{5}$ of the spacing between centers of the troughs, then Tables VI and VII apply directly. For other spacings, similar tables can be obtained by use of Eqs. (18) and (19).

In order to harmonize Section 7 with Section 6, the ceiling reflectances ρ_c were chosen so that $\rho_2 = 0.80$, 0.70, and 0.50 when ρ_p was 0.50. According to Eq. (16), this choice requires that the true reflectance of ceiling between panels shall be $\rho_c = 0.875$, 0.750, and 0.50.

As stated in Section 3, all the other quantities remain the same as in lighting from the complete ceiling. Thus Tables I, II, III, and V are used for panel lighting as well as for indirect lighting.

8. WALL LIGHTING, TYPE Ib.

In wall lighting, the luminaires are arranged to light the walls with no spill light to the ceiling or floor. Equations (1) to (12) apply. They allow the tabulation of interflectance and helios on the basis of the previous tables.[2] The new tables, VIII to XII, present calculated values of f_1, H_{IT}/D_{AV}, H_{IM}/D_{AV}, H_2/D_{AV}, and H_3/D_{AV} for wall lighting.

9. FLOOR LIGHTING, TYPE IIIb.

With IIIb lighting, luminaires are built into the ceiling or hung below it and are so louvered that all of their luminous output is incident on the floor. The application of Eqs. (26) to (28) to the data of the previous paper [2] gives the new Tables XIII to XVII.

10. WALL-AND-CEILING LIGHTING, TYPE Ib, IIb.

All lighting systems can be calculated as combinations of the preceding basic systems. Sections 4 and 5 indicate how this can be accomplished by the method of superposition. An example is now considered, the room being lighted by luminaires that produce the same uniform incident pharosage on the ceiling and on the walls, no direct light being sent to the floor. If D_{01} is the incident pharosage on the walls, obtained directly from the luminaires, and D_{02} is the incident pharosage on the ceiling, then

$$D_{01} = D_{02}$$

and

$$\frac{F_1}{F_D} = \frac{D_{01}S_1}{F_D} = \frac{D_{01}}{F_D}\left(\frac{S_1}{S}\right)S = 4k_r\frac{D_{01}S}{F_D}$$

$$\frac{F_2}{F_D} = \frac{D_{02}S}{F_D}.$$

But

$$F_D = F_1 + F_2,$$

so

$$\frac{F_1}{F_D} = \frac{4k_r}{1 + 4k_r}, \qquad \frac{F_2}{F_D} = \frac{1}{1 + 4k_r}. \tag{35}$$

These pharos ratios were employed in Eqs. (3) to (32) to give the data of Tables XVIII to XXII.

11. DIFFUSING-GLOBE LIGHTING TYPE Ib, IIb, IIIb.

An inexpensive type of lighting that is still widely used employs incandescent lamps with enclosing globes of opal glass. The luminous output of the luminaires reaches ceiling, walls, and floor. The exact values of the pharos ratios F_1/F_D, F_2/F_D, F_3/F_D depend to some extent

on the shape of the globes and on their mounting height. The method of Section 5 allows the exact ratios to be determined for any given condition. For practical purposes, however, it suffices to take a representative case. Spherical luminaires of uniform helios were assumed and the pharos ratios were calculated by means of Eq. (34). These ratios were then substituted in Eqs. (30) to (32) to obtain the data of Tables XII to XXVII.

Figure 4 shows how the interflectance curves differ for various kinds of lighting. All graphs refer to rooms having white ceilings ($\rho_2 = 0.80$),

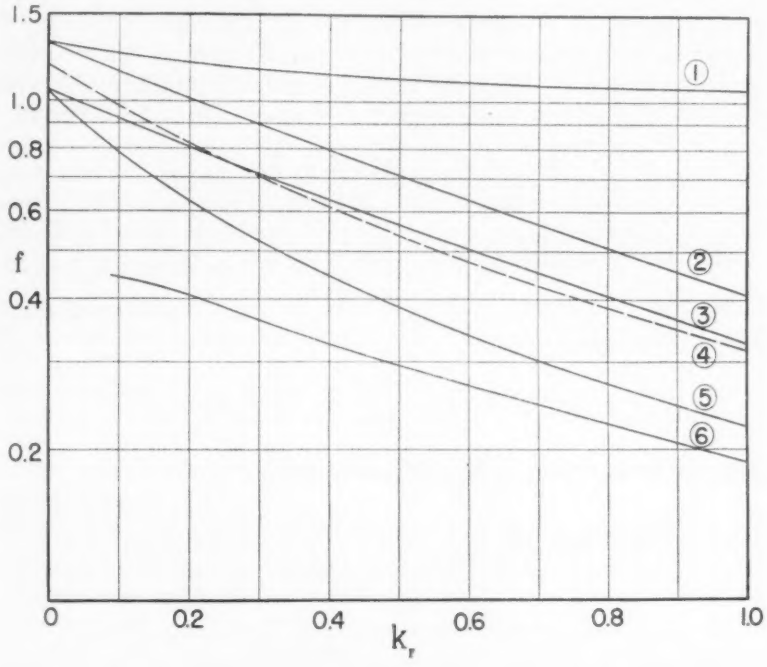

FIG. 4. Interflectance curves for various kinds of lighting. Reflectances are $\rho_1 = 0.50$, $\rho_2 = 0.80$, $\rho_3 = 0.30$. 1. Floor lighting, Type IIIb. 2. Lighting from ceiling panels, Type IIa. 3. Ceiling lighting, Type II b. 4. Diffusing-globe lighting, Type Ib, IIb, IIIb. 5. Wall-and-ceiling lighting, Type Ib, IIb. 6. Wall lighting, Type Ib.

walls with reflectance 0.50, and floors with reflectance 0.30. Evidently direct lighting gives highest interflectance, particularly for small rooms (k_r large). This advantage, however, is more than balanced for most applications by the disadvantage of shadows and glare that are associated with direct lighting. The curves for panel lighting and indirect lighting are identical except for a factor of 0.80 (ceiling reflectance) that enters in the case of indirect lighting. Lowest interflectances are obtained with wall lighting. In comparing the curves for economy of the various types of lighting, one should remember that Fig. 4 refers only

to one particular combination of reflectances. Also, the logance of the luminaires is an important consideration which is not included in Fig. 4. For instance, the curve for panel lighting is higher than the curve for indirect lighting; but the coefficient of utilization ($k_u = fg$) may be higher for indirect lighting because of the low values of logance usually obtained in panel lighting.

12. EXAMPLE.

As an example,[6] consider the lighting of a private office 4.0 × 5.0 m. with a 3.11 m. ceiling (13.1 × 16.4 × 10.2 ft.). It is proposed to use indirect lighting with a white ceiling ($\rho_2 = 0.80$), walnut-paneled walls ($\rho_1 = 0.10$), a walnut desk ($\rho_D = 0.10$), and dark floor ($\rho_3 = 0.10$).

The total phraos F_L from the lamps is to be 16,660 lumens, and the logance of the luminaires is to be 0.75. The domance k_r of the room is

$$k_r = \frac{h(w+l)}{2wl} = \frac{3.11(4+5)}{2(4)(5)} = 0.70.$$

From Table I, the interflectance is found to be

$$f = 0.3034$$

and thus, by Eq. (1),

$$D_{AV} = 0.3034(0.75)\frac{16660}{20} = 190 \text{ lumens m.}^{-2}.$$

Similarly, from Tables II to V one obtaius

Walls, near ceiling	$H_{1T}/D_{AV} = 0.1395$,	$H_{1T} = 26.4$ blondels;
Walls, midway between ceiling and floor	$H_{1M}/D_{AV} = 0.0728$,	$H_{1M} = 13.8$;
Ceiling	$H_2/D_{AV} = 2.705$,	$H_2 = 514$;
Floor	$H_3/D_{AV} = 0.0718$,	$H_3 = 13.6$;
Desk top	190(0.10),	$H_D = 19.0$;
Printed paper on desk	190(0.65),	$H_A = 123$.

This example indicates the simplicity of the method and shows the large amount of useful information that can be obtained in a few minutes.[6] Any of the other types of lighting considered in Sections 6 to 10 could be handled with no more trouble. Only in the case of luminaires that do not fit into any of the foregoing categories is it necessary to go back to the superposition method of Section 4 and do any appreciable amount of calculation.

13. SATISFACTORY AND UNSATISFACTORY LIGHTING.

The principal fault of most artificial lighting installations is the great range of helios, a range that is far in excess of that usually ob-

[6] Parry Moon and D. E. Spencer, "A Simple Criterion for Quality in Lighting," *Illum. Eng.* (1946).

tained outdoors. It is the large variation of helios in the visual field that is responsible for much of the eyestrain encountered today. In the example of Section 12, for instance, if a person is reading at the desk, his eyes adapt themselves to the printed paper. An average reflectance of printed paper is 0.65, so the *adaptation helios* is

$$H_A = 190(0.65) = 123 \text{ blondels.}$$

Numerous tests [7] indicate that best vision will be obtained when the whole visual field has approximately this helios. A useful criterion for close visual tasks extending over long periods of time, however, is that the adaptation helios for any part of the room *shall not exceed 3 times the adaptation helios for the work* and shall be at least $\frac{1}{3}$ the adaptation helios for the work. In practice, this statement is usually approximated by saying $H_{max}/H_A = 3$ when H_{max} is the highest helios of an extended surface, and $H_A/H_{min} = 3$ where H_{min} is the lowest helios of an extended surface. If the surfaces are very small, larger ratios may be used, the exact value being obtained by calculating the adaptation helios.[8]

Ratios obtained in the foregoing example are

Walls, near ceiling	$H_A/H_{1T} = 4.65$
Walls, midway between ceiling and floor	$H_A/H_{1M} = 8.90$
Ceiling	$H_2/H_A = 4.16$
Floor	$H_A/H_3 = 9.00$
Desk	$H_A/H_D = 6.50$

All these values are too large and show that the proposed design would be unsatisfactory.

The tables of the present paper also allow a general analysis of lighting, to indicate what range of reflectances and what types of lighting satisfy the 3:1 criterion. Figures 5 to 10 are made with the same general form as the tables. Whenever all the helios retios satisfy the 3:1 criterion, the rectangle is left blank. If one of the helios ratios, for a given set of reflectances, exceeds 3 but is less than 10, the area is cross-hatched. If some ratio exceeds 10, however, the area is made black. *Figures 5 to 10 apply only to cases where the eyes are adapted to printed paper having an average reflectance of 0.65.*

The diagrams give practical conclusions at a glance. Consider indirect lighting, for example. Figure 5 shows that for a floor reflectance of 0.10, the 3:1 criterion is never satisfied. Even with a floor reflectance of 0.30, indirect lighting is unsatisfactory for small rooms and with wall reflectances of 0.30 or less. One may conclude

[7] Parry Moon and D. E. Spencer, "The Visual Effect of Nonuniform Surrounds," *J. Opt. Soc. Am.*, **35**, 233 (1945).

[8] Parry Moon and D. E. Spencer, "The Specification of Foveal Adaptation," *J. Opt. Soc. Am.*, **33**, 444 (1943).

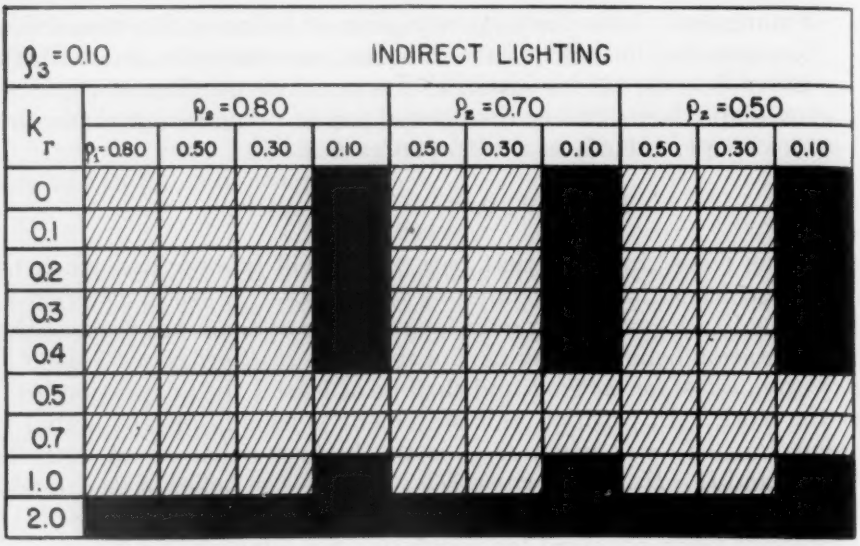

FIG. 5a.

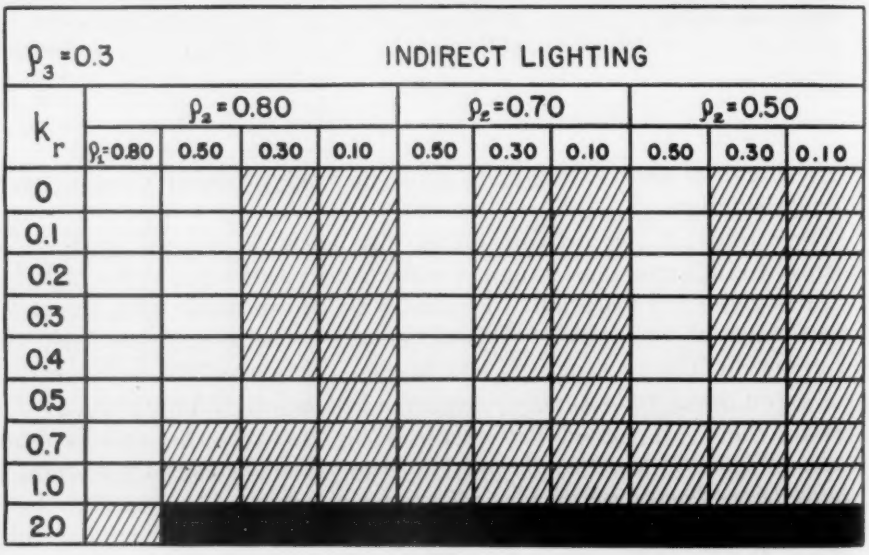

FIG. 5b.

FIG. 5. Analysis chart for ceiling lighting, Type IIb. ρ_1 = average wall reflectance; ρ_2 = average ceiling reflectance; ρ_3 = average floor reflectance. The domance k_r specifies the shape of the room and is calculated by means of the equation,

$$k_r = \frac{h(w + l)}{2wl}.$$

Blank areas of the chart indicate that helios ratios H_i/H_A and H_A/H_i are all less than 3. Cross-hatched areas indicate that the highest ratio is between 3 and 10. Black areas indicate that the highest ratio is greater than 10.

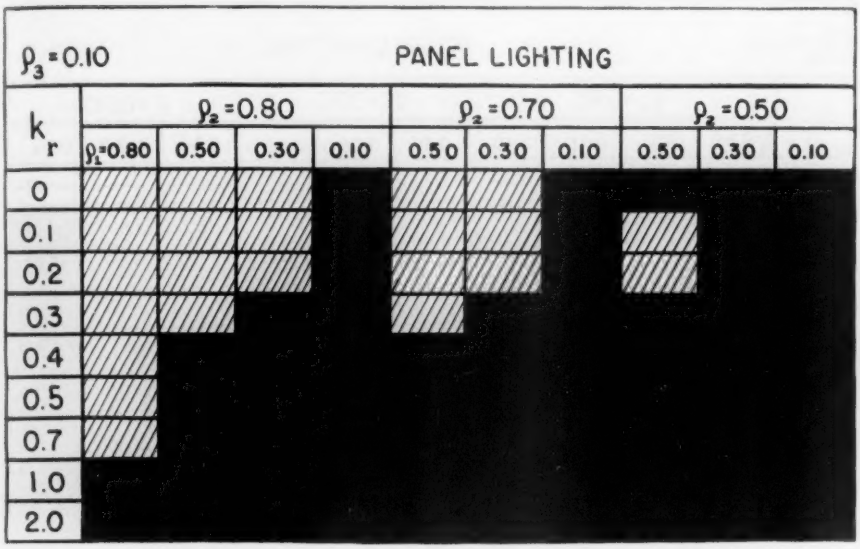

FIG. 6a.

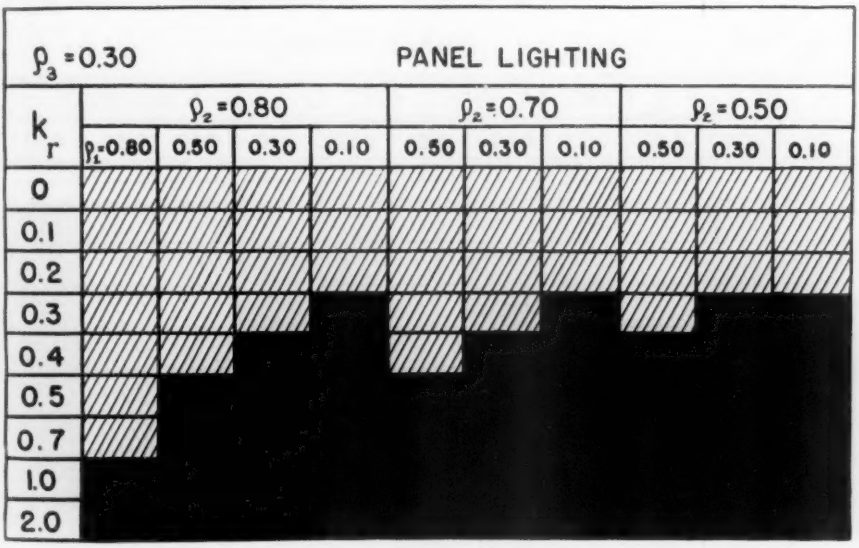

FIG. 6b.

FIG. 6. Analysis chart for panel lighting, Type IIa.

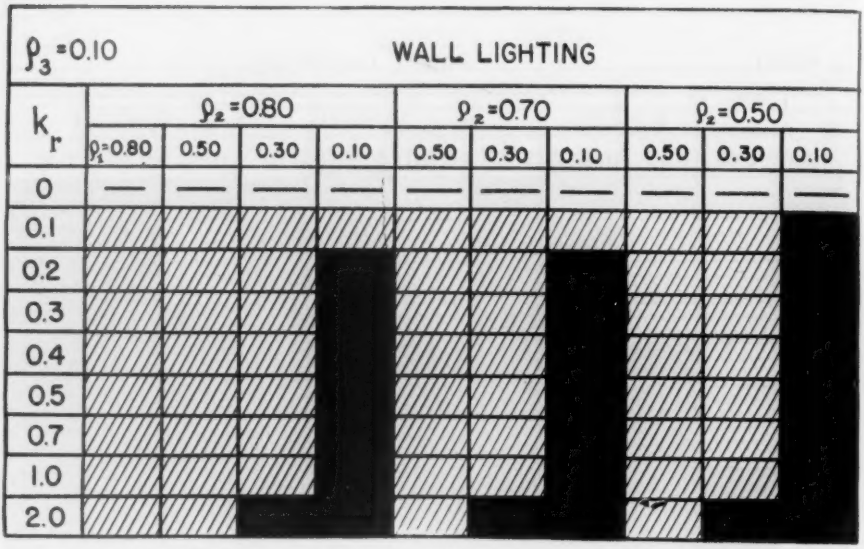

FIG. 7a.

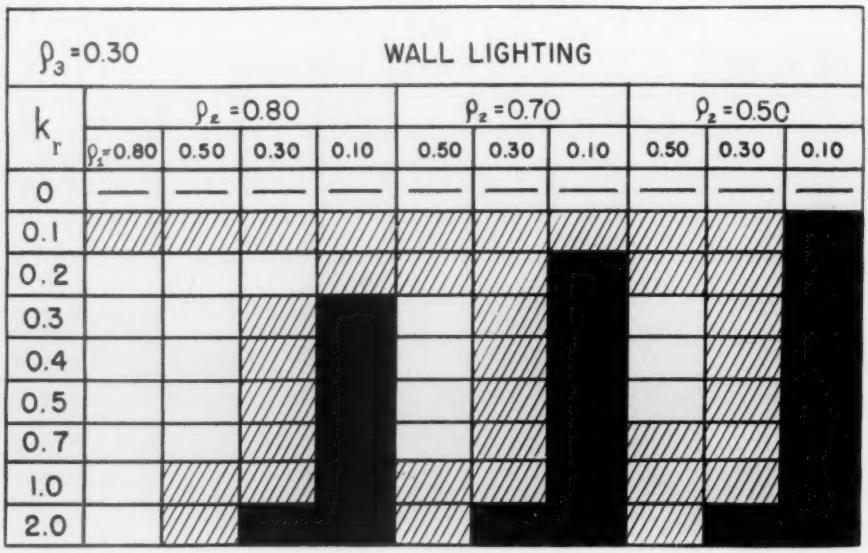

FIG. 7b.

FIG. 7. Analysis chart for wall lighting, Type Ib.

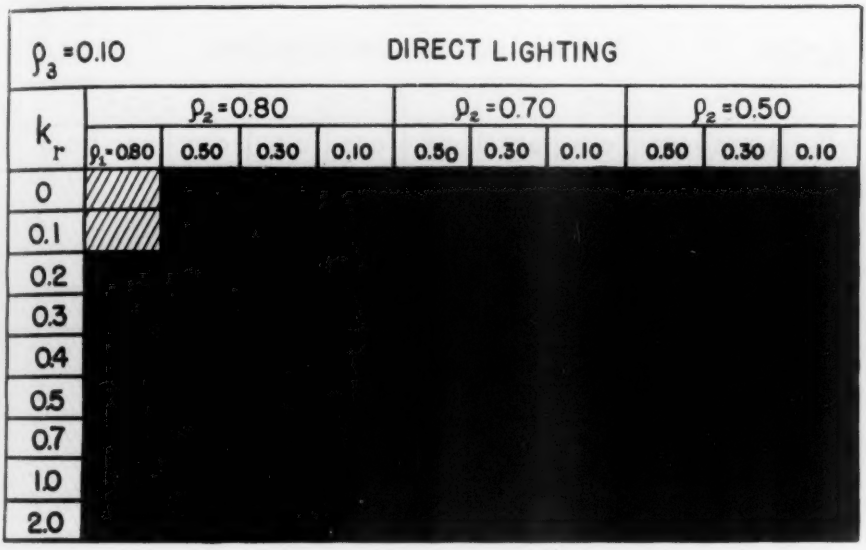

FIG. 8a.

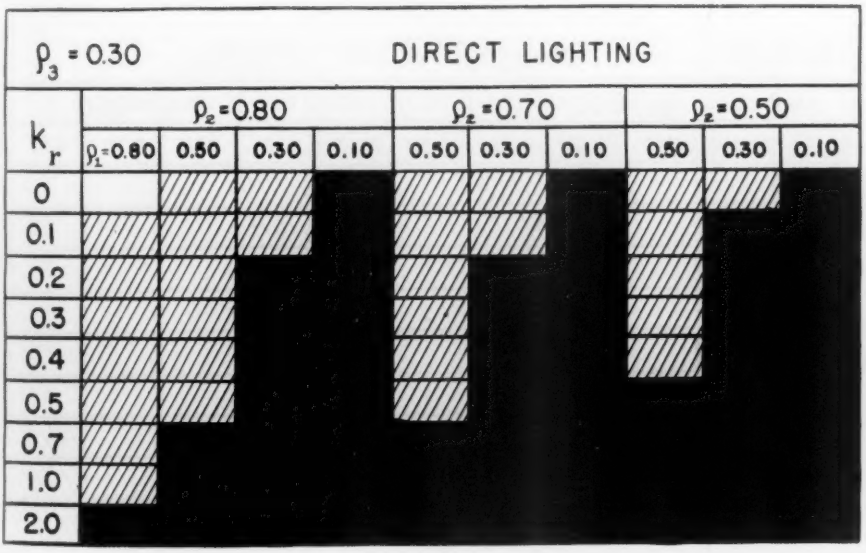

FIG. 8b.

FIG. 8. Analysis chart for floor lighting, Type IIIb.

ρ_3 = 0.10	WALL-AND-CEILING LIGHTING									
k_r	ρ_2 = 0.80				ρ_2 = 0.70			ρ_2 = 0.50		
	ρ_1 = 0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
0										
0.1										
0.2										
0.3										
0.4										
0.5										
0.7										
1.0										
2.0										

FIG. 9a.

ρ_3 = 0.30	WALL-AND-CEILING LIGHTING									
k_r	ρ_2 = 0.80				ρ_2 = 0.70			ρ_2 = 0.50		
	ρ_1 = 0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
0										
0.1										
0.2										
0.3										
0.4										
0.5										
0.7										
1.0										
2.0										

FIG. 9b.

FIG. 9. Analysis chart for wall-and-ceiling lighting, Type Ib, IIb.

ρ_3 = 0.10	DIFFUSING-GLOBE LIGHTING									
k_r	ρ_2 = 0.80				ρ_2 = 0.70			ρ_2 = 0.50		
	ρ_1 = 0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
0										
0.1										
0.2										
0.3										
0.4										
0.5										
0.7										
1.0										
2.0										

FIG. 10a.

ρ_3 = 0.30	DIFFUSING-GLOBE LIGHTING									
k_r	ρ_2 = 0.80				ρ_2 = 0.70			ρ_2 = 0.50		
	ρ_1 = 0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
0										
0.1										
0.2										
0.3										
0.4										
0.5										
0.7										
1.0										
2.0										

FIG. 10b.

FIG. 10. Analysis chart for lighting from uniformly diffusing globes, Type Ib, IIb, IIIb.

that in general *indirect lighting is satisfactory only if the reflectance of the floor is at least 0.30 and the average reflectance of the walls at least 0.50.*

Panel lighting (Fig. 6) encounters the difficulties of indirect lighting and in addition suffers from excessive helios of the panels and insufficient helios of the ceiling between panels. As a result, the 3:1 criterion cannot be satisfied under any conditions. This conclusion applies, of course, only to the reflectances and panel areas assumed in computing Tables VII and VIII.

Figure 7 shows that wall lighting is sometimes satisfactory if floor reflectance is at least 0.30 and wall reflectance at least 0.50. Direct lighting (Fig. 8) is seen to be unsatisfactory in nearly all cases, even assuming (as was done in making the diagrams) that the luminaires are so well shielded that they do not cause direct glare.

Figure 9 gives results for a combination of wall lighting and ceiling lighting. This type of lighting tends to give better results than any of the previous cases. *It is satisfactory in all shapes of rooms if the floor reflectance is at least 0.30 and the wall reflectance at least 0.50.* A similar conclusion applies with diffusing globes except that the globes themselves are generally potent sources of glare. The helios of the globes has not been considered in Fig. 10 since it depends on additional factors.

It should be emphasized that the foregoing conclusions apply only when the observer's eyes are adapted to white paper on a horizontal surface. Figures 5 to 10 apply to the lighting of offices, drafting rooms, class rooms, and libraries; but they do not apply to such operations as machining bakelite, inspecting leather, or sewing on dark cloth. In such cases, the room reflectances must be much lower to prevent excessive contrasts and the type of lighting may have to be radically modified.

14. SUMMARY.

In the conventional method of lighting design, attention has been confined to the incident pharosage (lumen m^{-2}) on the work and no consideration has been given to the important question of helios variation in the visual field. The solution of an integral equation made possible the calculation of these values of helios which were tabulated in a previous paper [2] for three basic types of lighting.

The present paper extends these tables to cover a number of other types of lighting. Tabulated values and graphs are presented so that most lighting design can be accomplished very expeditiously. For other types of luminaires, not considered in the tables, the principle of superposition allows the engineer to make similar tabulations. The necessary equations are given in the paper.

As an example of the use of the tables, charts were prepared to show what reflectances and room shapes allow satisfactory lighting. It is shown that in many cases the visual field can be made reasonably uni-

form, so that it satisfies a 3:1 helios criterion. For indirect lighting and for a combination of wall and ceiling lighting, the criterion can be satisfied usually if the floor reflectance is at least 0.30 and the average wall reflectance at least 0.50. For direct lighting and for lighting from troffers and panels, however, a 3:1 ratio is almost impossible. The prevalent use of direct fluorescent lighting and low reflectances (0.02 to 0.15) is undoubtedly responsible for much of the glare and the visual fatigue associated with artificial lighting today.

APPENDIX.

List of Symbols.

D_{AV} = average pharosage (lumens m^{-2}) incident on the upper side of an imaginary horizontal surface at table level.

$(D_{AV})_1$, $(D_{AV})_2$, $(D_{AV})_3$ = average pharosage (including interflections) produced by wall source, ceiling source, and floor source, respectively.

$f = F_W/F_D$ = interflectance (numeric) of the room.

f_1, f_2, f_3 = interflectance for wall source, ceiling source, and floor source, respectively.

F_D = total pharos (lumen) from luminaires.

F_L = total pharos from lamps.

F_W = total pharos incident on the upper side of an imaginary horizontal surface at table level.

F_1, F_2, F_3 = pharos from luminaires to walls, ceiling, and floor, respectively:

$$F_D = F_1 + F_2 + F_3.$$

$g = F_D/F_L$ = logance (numeric) of the luminaires.

h = distance (m) between ceiling and floor of the room.

H_A = adaptation helios (blondel). For a large uniform surface of helios H, the eyes adapt to this value and the adaptation helios is said to be $H_A = H$.

H_D = helios (blondel) of desk.

H_{1M} = average helios of walls on a line midway between ceiling and floor.

H_{1T} = average helios of walls, near ceiling.

H_2 = average helios of ceiling.

H_3 = average helios of floor.

H_{01}, H_{02}, H_{03} = helios of walls, ceiling, and floor, caused by direct light from the luminaires (no interflections).

$k_r = \dfrac{h(l+w)}{2lw}$ = domance (numeric).

$k_u = fg$ = coefficient of utilization (numeric).

l = length (m) of room.

S = area (m^2) of floor.

S_1 = total area (m^2) of walls.

S_p = total area (m^2) of luminous panels.

w = width (m) of room.

ρ_1, ρ_2, ρ_3 = average reflectance (numeric) of walls, ceiling, and floor, respectively.

ρ_c = reflectance of ceiling between luminous panels.

ρ_p = reflectance of luminous panels.

ρ_w = reflectance of wall between luminous panels.

TABLE I.

Indirect Lighting, f.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
					ρ_3 = 0.10 (Floor)					
0	08695	08695	08695	08695	07527	07527	07527	05263	05263	05263
0.1	8382	7925	7650	7396	6845	6625	6419	4767	4636	4512
0.2	8049	7202	6736	6331	6210	5835	5505	4312	4085	3883
0.3	7702	6528	5934	5442	5622	5141	4737	3894	3600	3349
0.4	7350	5907	5229	4691	5083	4530	4087	3514	3174	2894
0.5	6996	5339	4609	4052	4590	3994	3532	3169	2798	2504
0.7	6296	4348	3582	3034	3735	3105	2646	2574	2176	1878
1.0	5311	3158	02478	01974	02729	02129	01722	01878	01493	01223
2.0	2853	01107	17001	14746	19497	16068	14142	16529	14254	12943
					ρ_3=0.30 (Floor)					
0	1.0526	1.0526	1.0526	1.0526	08861	08861	08861	05883	05883	05883
0.1	09946	09204	08774	08386	7786	7455	7153	5216	5034	4865
0.2	9382	8040	7443	6886	6882	6363	5922	4641	4345	4089
0.3	8842	7174	6388	5764	6108	5484	4981	4174	3774	3474
0.4	8322	6374	5526	4882	5439	4757	4232	3699	3292	2971
0.5	7826	5680	4806	4166	4852	4146	3620	3309	2880	2553
0.7	6907	4538	3674	3076	3884	3177	2679	2658	2217	1902
1.0	5703	3265	02490	01985	02798	02157	01731	01920	01509	01236
2.0	2957	01117	17026	14750	19583	16089	14145	16588	14269	12945

NOTE—Superscripts refer to number of zeros after decimal point. When no superscript is used, a zero is understood. For example, 1906 = 0.1906, $^{1}6335$ = 0.06335.

TABLE II.

Indirect Lighting, H_{1T}/D_{AV}.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10	ρ_1=0.50	ρ_1=0.30	ρ_1=0.10
					ρ_3=0.10 (Floor)					
0	04400	02750	01650	15500	02750	01650	15500	02750	01650	15500
0.1	4997	3114	1865	16204	3114	1864	16204	3114	1864	16204
0.2	5599	3509	2109	17045	3509	2109	17044	3509	2109	17044
0.3	6209	3941	2388	18038	3941	2387	18038	3940	2387	18038
0.4	6834	4413	2704	19201	4413	2704	19200	4413	2704	19201
0.5	7473	4932	3063	01056	4932	3063	01056	4932	3063	01056
0.7	08822	6138	3934	1395	6138	3934	1395	6138	3933	1395
1.0	1.1075	08482	05728	2131	08481	05728	2131	08479	05728	2131
2.0	2.203	2.453	2.010	8835	2.452	2.009	8835	2.452	2.008	8835
					ρ_3=0.30 (Floor)					
0	05200	03250	01950	16500	03249	01950	16500	03249	01950	16500
0.1	5645	3513	2102	16988	3513	2102	16987	3513	2102	16986
0.2	6111	3823	2294	17658	3823	2294	17656	3823	2294	17654
0.3	6599	4181	2530	18512	4181	2530	18512	4181	2530	18507
0.4	7112	4593	2813	19567	4592	2812	19566	4592	2812	19558
0.5	7654	5060	3143	01084	5060	3143	01084	5060	3143	01082
0.7	08835	6181	3971	1411	6181	3971	1411	6181	3971	1407
1.0	1.089	08434	05725	2137	08434	05725	2136	08434	05725	2123
2.0	2.145	2.433	2.002	08831	2.433	2.002	08829	2.433	2.002	08829

TABLE III.

Indirect Lighting, H_{1M}/D_{AV}

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	04400	02750	01650	15500	02750	01650	15500	02750	01650	15500
0.1	4634	2876	1717	15699	2876	1717	15699	2876	1717	15699
0.2	4861	3003	1788	15917	3003	1788	15917	3003	1788	15917
0.3	5084	3133	1863	16155	3133	1863	16154	3133	1863	16154
0.4	5303	3266	1940	16410	3266	1940	16409	3266	1940	16409
0.5	5517	3400	2022	16683	3400	2022	16682	3400	2022	16682
0.7	5937	3677	2195	17284	3677	2195	17284	3677	2195	17284
1.0	6553	4125	2486	18329	4125	2486	18329	4125	2486	18327
2.0	8637	5941	3773	01327	5941	3773	01327	5941	3773	01327
					$\rho_3=0.30$ (Floor)					
0	05200	03250	01950	16500	03250	01950	16500	03249	01950	16500
0.1	5356	3319	1980	16566	3319	1980	16565	3319	1980	16564
0.2	5512	3396	2018	16668	3396	2018	16666	3396	2018	16665
0.3	5668	3480	2064	16802	3480	2064	16802	3480	2064	16798
0.4	5826	3571	2115	16969	3571	2115	16969	3571	2115	16963
0.5	5984	3670	2175	17167	3670	2175	17166	3670	2175	17156
0.7	6307	3885	2311	17648	3884	2311	17645	3884	2311	17622
1.0	6806	4261	2561	18561	4261	2561	18560	4261	2561	18507
2.0	8655	5962	3787	01332	5961	3784	01332	5961	3784	01332

TABLE IV.

Indirect Lighting, H_1/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0855	1.1144	1.1335	1.1525	1.1144	1.1335	1.1524	1.1144	1.1335	1.1523
0.2	1.1743	1.2410	1.2849	1.3282	1.2410	1.2848	1.3282	1.2410	1.2848	1.3282
0.3	1.2669	1.3814	1.4566	1.5313	1.3814	1.4566	1.5312	1.3814	1.4566	1.5312
0.4	1.3639	1.5371	1.6513	1.7653	1.5371	1.6513	1.7651	1.5371	1.6513	1.7652
0.5	1.4656	1.7096	1.8720	2.035	1.7096	1.8720	2.035	1.7096	1.8720	2.035
0.7	1.6861	2.113	2.406	2.705	2.113	2.406	2.705	2.113	2.406	2.705
1.0	2.067	2.906	3.506	4.146	2.906	3.506	4.146	2.905	3.506	4.146
2.0	4.005	8.379	12.301	17.218	8.375	12.301	17.218	8.375	12.301	17.218
					$\rho_3=0.30$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0741	1.1075	1.1293	1.1510	1.1075	1.1293	1.1510	1.1075	1.1293	1.1509
0.2	1.1526	1.2280	1.2770	1.3262	1.2279	1.2770	1.3257	1.2279	1.2770	1.3255
0.3	1.2359	1.3629	1.4457	1.5278	1.3628	1.4457	1.5278	1.3628	1.4457	1.5278
0.4	1.3242	1.5138	1.6379	1.7608	1.5137	1.6377	1.7608	1.5135	1.6377	1.7594
0.5	1.4183	1.6821	1.8561	2.030	1.6820	1.8561	2.030	1.6820	1.8559	2.027
0.7	1.6252	2.078	2.386	2.699	2.078	2.386	2.699	2.078	2.386	2.691
1.0	1.9893	2.861	3.481	4.138	2.861	3.481	4.138	2.861	3.481	4.112
2.0	3.885	8.307	12.260	17.204	8.304	12.260	17.204	8.304	12.259	17.204

TABLE V.

Indirect Lighting, H_3/D_{AV}.

k_r	$\rho_1=0.80$ (Ceiling)				$\rho_1=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.9725	1.9644	1.9592	1.9540	1.9644	1.9591	1.9540	1.9644	1.9591	1.9540
0.2	1.9461	1.9302	1.9198	1.9097	1.9302	1.9198	1.9097	1.9301	1.9198	1.9097
0.3	1.9205	1.8973	1.8821	1.8680	1.8972	1.8821	1.8678	1.8972	1.8821	1.8678
0.4	1.8958	1.8655	1.8461	1.8277	1.8655	1.8461	1.8277	1.8655	1.8461	1.8277
0.5	1.8720	1.8349	1.8115	1.7895	1.8349	1.8115	1.7894	1.8349	1.8115	1.7894
0.7	1.8271	1.7768	1.7465	1.7180	1.7768	1.7465	1.7180	1.7768	1.7465	1.7180
1.0	1.7649	1.6982	1.6585	1.6230	1.6980	1.6585	1.6229	1.6980	1.6585	1.6229
2.0	1.5965	1.4889	1.4335	1.3880	1.4889	1.4335	1.3880	1.4889	1.4335	1.3881
					$\rho_3=0.30$ (Floor)					
0	.3000	.3000	.3000	.3000	.3000	.3000	.3000	.3000	.3000	.3000
0.1	2929	2901	2882	2863	2901	2882	2863	2901	2882	2863
0.2	2860	2804	2767	2733	2804	2767	2732	2803	2767	2732
0.3	2792	2709	2657	2607	2709	2657	2606	2709	2657	2605
0.4	2726	2619	2551	2487	2619	2551	2487	2619	2551	2484
0.5	2663	2532	2449	2373	2531	2449	2373	2531	2449	2369
0.7	2540	2363	2258	2160	2363	2257	2160	2362	2257	2153
1.0	2368	2132	1996	1875	2132	1996	1875	2132	1996	1863
2.0	1882	1507	1320	1169	1507	1320	1169	1506	1320	1169

TABLE VI.

Troffer Lighting, f.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	1.0869	1.0869	1.0869	1.0869	1.0753	1.0753	1.0753	1.0526	1.0526	1.0526
0.1	1.0478	.9906	.9563	.9245	.9779	.9464	.9170	.9534	.9271	.9024
0.2	1.0061	9002	8420	7914	8872	8335	7864	8623	8169	7765
0.3	.9628	8160	7417	6802	8032	7344	6767	7788	7200	6698
0.4	9188	7384	6536	5864	7262	6472	5839	7028	6347	5788
0.5	8745	6674	5761	5065	6557	5705	5046	6338	5596	5007
0.7	7870	5435	4478	3792	5336	4435	3780	5148	4352	3755
1.0	6639	3973	.3072	.2467	3898	.3042	.2460	3756	.2985	.2446
2.0	.3566	.1384	1.8751	1.5933	.1357	1.8668	1.5917	.1306	1.8507	1.5885
					$\rho_3=0.30$ (Floor)					
0	1.3158	1.3158	1.3158	1.3158	1.2658	1.2658	1.2658	1.1765	1.1765	1.1765
0.1	1.2432	1.1505	1.0968	1.0483	1.1124	1.0650	1.0219	1.0432	1.0067	.9730
0.2	1.1728	1.0131	.9304	.8607	.9831	.9090	.8460	.9281	.8690	8178
0.3	1.1052	.8968	7985	7205	8726	7834	7115	8278	7548	6947
0.4	1.0403	7968	6907	6102	7770	6796	6046	7397	6583	5941
0.5	.9783	7100	6008	5207	6932	5923	5171	6618	5760	5106
0.7	8634	5672	4593	3845	5548	4539	3827	5316	4434	3803
1.0	7129	4081	.3113	.2481	3997	.3081	.2473	3840	.3018	.2472
2.0	.3696	.1396	1.8783	1.5938	.1369	1.8699	1.5922	.1318	1.8537	1.5890

TABLE VII

Troffer Lighting, H_T/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
				$\rho_3=0.10$ (Floor)						
0	4.651	4.651	4.651	4.651	4.700	4.700	4.700	4.800	4.800	4.800
0.1	4.854	5.113	5.283	5.453	5.179	5.338	5.497	5.310	5.448	5.585
0.2	5.082	5.636	5.999	6.358	5.717	6.060	6.399	5.880	6.181	6.480
0.3	5.336	6.225	6.809	7.389	6.323	6.876	7.442	6.518	7.012	7.503
0.4	5.614	6.886	7.726	8.564	6.999	7.801	8.601	7.229	7.954	8.676
0.5	5.919	7.624	8.764	9.910	7.757	8.849	9.947	8.021	9.020	10.024
0.7	6.613	9.371	11.339	13.254	9.541	11.382	13.270	9.88	11.597	13.358
1.0	7.882	12.828	16.526	20.33	13.070	16.593	20.38	13.555	16.907	20.50
2.0	14.772	36.86	57.68	84.50	37.57	58.23	84.73	39.01	59.32	85.19
				$\rho_3=0.30$ (Floor)						
0	3.950	3.950	3.950	3.950	4.100	4.100	4.100	4.400	4.400	4.400
0.1	4.191	4.495	4.695	4.893	4.644	4.831	5.016	4.942	5.103	5.262
0.2	4.451	5.127	5.500	5.912	5.236	5.627	6.013	5.538	5.880	6.217
0.3	4.731	5.730	6.352	7.027	5.885	6.503	7.115	6.143	6.745	7.285
0.4	5.033	6.437	7.358	8.270	6.597	7.476	8.346	6.921	7.714	8.492
0.5	5.359	7.316	8.442	9.671	7.384	8.562	9.738	7.726	8.800	9.861
0.7	6.083	9.012	11.017	13.066	9.209	11.147	13.127	9.603	11.407	13.209
1.0	7.380	12.508	16.230	20.22	12.766	16.396	20.28	13.277	16.735	20.29
2.0	14.265	36.52	57.47	84.43	37.24	58.02	84.66	38.66	59.11	85.12

TABLE VIII.

Troffer Lighting, H_c/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
				$\rho_3=0.10$ (Floor)						
0	.18755	.18755	.18755	.18755	.17503	.17503	.17503	.14997	.14997	.14997
0.1	.01434	.01148	.09607	.17749	.19839	.18230	.16637	.16566	.15490	.14422
0.2	.01973	1424	.01064	.17076	.01221	.19114	.16063	.18141	.16072	.14044
0.3	2497	1706	1186	.16689	14620	.01017	.15731	.19746	.16778	.13822
0.4	3014	2000	1327	.16565	17144	1137	.15633	.01143	.17578	.13749
0.5	3523	2311	1490	.16651	19788	1277	.15712	1319	.18524	.13815
0.7	4545	2992	1895	.17442	2564	1623	.16406	1709	.01082	.14261
1.0	.06137	.04253	2738	.01015	.03648	2349	.18711	2431	1565	.15805
2.0	1.3130	1.2589	9561	3964	1.0756	8207	.03404	7168	5466	.02277
				$\rho_3=0.30$ (Floor)						
0	.02625	.02625	.02625	.02625	.02250	.02250	.02250	.01500	.01500	.01500
0.1	2950	2606	2379	2156	2234	2040	1847	1489	1360	1232
0.2	3281	2657	2212	1798	2258	1896	1540	1505	1265	.01027
0.3	3621	2711	2115	1530	2323	1814	1312	1536	1210	.18738
0.4	3970	2830	2079	1335	2430	1781	1145	1617	1188	.17625
0.5	4333	2993	2097	1202	2566	1798	.01030	1710	1198	.16855
0.7	5108	3450	2288	1078	2957	1962	.19240	1971	1308	.16153
1.0	.06416	.04489	2937	1177	.03847	2518	.01009	2565	1680	.16675
2.0	1.2903	1.2532	9564	3979	1.0706	8191	3419	7135	5459	.02275

TABLE IX.

Wall Lighting, f.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	—	—	—	—	—	—	—	—	—	—
0.1	$^{0}6608$	$^{0}3884$	$^{0}2238$	$^{1}7180$	$^{0}3604$	$^{0}2083$	$^{1}6695$	$^{0}3068$	$^{0}1781$	$^{0}5753$
0.2	6394	3540	1973	$^{1}6140$	3299	1847	$^{1}5770$	2838	1602	$^{1}5043$
0.3	6174	3231	1752	$^{1}5320$	3029	1649	$^{1}5032$	2634	1450	$^{1}4467$
0.4	5953	2966	1567	$^{1}4663$	2788	1483	$^{1}4438$	2451	1321	$^{1}3994$
0.5	5736	2728	1410	$^{1}4128$	2576	1342	$^{1}3951$	2287	1209	$^{1}3600$
0.7	5320	2330	$^{0}1165$	$^{1}3321$	2219	$^{0}1119$	$^{1}3209$	2008	$^{0}1029$	$^{1}2987$
1.0	4752	1811	$^{1}9088$	$^{1}2528$	1744	$^{1}8828$	$^{1}2469$	1618	$^{1}8321$	$^{1}2354$
2.0	$^{0}3383$	$^{0}1113$	$^{1}5066$	$^{1}1358$	$^{0}1098$	$^{1}5018$	$^{1}1351$	$^{0}1069$	$^{1}4931$	$^{1}1335$
					$\rho_3=0.30$ (Floor)					
0	—	—	—	—	—	—	—	—	—	—
0.1	$^{0}8032$	$^{0}4427$	$^{0}2623$	$^{1}8313$	$^{0}4019$	$^{0}2392$	$^{1}7608$	$^{0}3416$	$^{0}1967$	$^{1}6303$
0.2	7624	4066	2222	$^{1}6796$	3727	2050	$^{1}6308$	3105	1730	$^{1}5380$
0.3	7241	3622	1917	$^{1}5715$	3348	1787	$^{1}5361$	2841	1540	$^{1}4680$
0.4	6887	3255	1679	$^{1}4907$	3031	1578	$^{1}4644$	2615	1385	$^{1}4131$
0.5	6552	2948	1489	$^{1}4285$	2763	1415	$^{1}4084$	2420	1257	$^{1}3692$
0.7	5951	2464	$^{0}1206$	$^{1}3390$	2337	$^{0}1155$	$^{1}3269$	2098	$^{0}1057$	$^{1}3030$
1.0	5196	1880	$^{1}9276$	$^{1}2551$	1808	$^{1}9001$	$^{1}2491$	1671	$^{1}8467$	$^{1}2370$
2.0	$^{0}3562$	$^{0}1133$	$^{1}5107$	$^{1}1361$	$^{0}1117$	$^{1}5062$	$^{1}1354$	$^{0}1087$	$^{1}4970$	$^{1}1338$

TABLE X.

Wall Lighting, H_{1T}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	—	—	—	—	—	—	—	—	—	—
0.1	3.555	3.549	3.549	3.548	3.801	3.801	3.801	4.410	4.413	4.413
0.2	2.115	2.110	2.108	2.105	2.239	2.237	2.235	2.546	2.547	2.548
0.3	1.6525	1.6483	1.6421	1.6381	1.7315	1.7295	1.7272	1.9322	1.9341	1.9354
0.4	1.4323	1.4244	1.4196	1.4147	1.4870	1.4843	1.4818	1.6310	1.6336	1.6363
0.5	1.3079	1.2987	1.2931	1.2879	1.3462	1.3435	1.3407	1.4538	1.4574	1.4612
0.7	1.1800	1.1685	1.1609	1.1556	1.1961	1.1925	1.1914	1.2571	1.2621	1.2696
1.0	1.1023	1.0871	1.0796	1.0737	1.0963	1.0945	1.0940	1.1155	1.1262	1.1373
2.0	1.0420	1.0161	1.0106	1.0099	$^{0}9962$	1.0023	1.0103	$^{0}9571$	$^{0}9856$	1.0119
					$\rho_3=0.30$ (Floor)					
0	—	—	—	—	—	—	—	—	—	—
0.1	3.086	3.084	3.084	3.082	3.361	3.361	3.361	4.039	4.042	4.043
0.2	1.9224	1.9191	1.9174	1.9157	2.059	2.059	2.058	2.395	2.398	2.400
0.3	1.5451	1.5410	1.5387	1.5364	1.6329	1.6326	1.6322	1.8495	1.8537	1.8573
0.4	1.3632	1.3588	1.3562	1.3535	1.4253	1.4250	1.4248	1.5788	1.5842	1.5898
0.5	1.2594	1.2542	1.2512	1.2483	1.3042	1.2990	1.3045	1.4175	1.4246	1.4315
0.7	1.1498	1.1438	1.1396	1.1376	1.1724	1.1724	1.1744	1.2354	1.2446	1.2560
1.0	1.0801	1.0721	1.0686	1.0666	1.0812	1.0838	1.0874	1.1005	1.1161	1.1317
2.0	1.0171	1.0044	1.0045	1.0081	$^{0}9846$	$^{0}9958$	1.0084	$^{0}9456$	$^{0}9794$	1.0097

TABLE XI.

Wall Lighting, H_{1M}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
				$\rho_3=0.10$ (Floor)						
0	—	—	—	—	—	—	—	—	—	—
0.1	3.550	3.546	3.548	3.547	3.799	3.799	3.799	4.414	4.414	4.413
0.2	2.105	2.104	2.104	2.104	2.235	2.235	2.235	2.550	2.550	2.549
0.3	1.6379	1.6371	1.6368	1.6358	1.7276	1.7271	1.7264	1.9397	1.9386	1.9369
0.4	1.4145	1.4136	1.4132	1.4127	1.4827	1.4819	1.4811	1.6421	1.6403	1.6386
0.5	1.2874	1.2866	1.2862	1.2856	1.3424	1.3414	1.3402	1.4689	1.4667	1.4643
0.7	1.1558	1.1554	1.1537	1.1535	1.1947	1.1922	1.1914	1.2814	1.2770	1.2746
1.0	1.0787	1.0763	1.0744	1.0723	1.1016	1.0986	1.0957	1.1552	1.1504	1.1455
2.0	1.0446	1.0282	1.0204	1.0137	1.0344	1.0268	1.0186	1.0463	1.0386	1.0291
				$\rho_3=0.30$ (Floor)						
0	—	—	—	—	—	—	—	—	—	—
0.1	3.083	3.082	3.082	3.082	3.361	3.361	3.361	4.044	4.044	4.044
0.2	1.9174	1.9160	1.9155	1.9152	2.060	2.059	2.058	2.404	2.403	2.402
0.3	1.5384	1.5369	1.5362	1.5355	1.6343	1.6335	1.6325	1.8637	1.8621	1.8602
0.4	1.3553	1.3541	1.3535	1.3528	1.4277	1.4265	1.4254	1.5981	1.5959	1.5936
0.5	1.2508	1.2493	1.2485	1.2475	1.3082	1.3070	1.3053	1.4419	1.4392	1.4364
0.7	1.1402	1.1395	1.1376	1.1370	1.1804	1.1776	1.1762	1.2705	1.2657	1.2629
1.0	1.0739	1.0710	1.0690	1.0670	1.0969	1.0939	1.0909	1.1516	1.1467	1.1419
2.0	1.0382	1.0248	1.0184	1.0129	1.0309	1.0240	1.0178	1.0430	1.0364	1.0281

TABLE XII.

Wall Lighting, H_2/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
				$\rho_3=0.10$ (Floor)						
0	—	—	—	—	—	—	—	—	—	—
0.1	0.5872	0.5875	0.5877	0.5877	0.5470	0.5470	0.5470	0.4474	0.4476	0.4476
0.2	6128	6132	6136	6138	5673	5677	5680	4578	4582	4584
0.3	6363	6371	6376	6380	5862	5868	5872	4669	4673	4676
0.4	6581	6592	6598	6605	6033	6040	6045	4746	4751	4757
0.5	6785	6794	6803	6810	6187	6195	6201	4810	4818	4824
0.7	7143	7147	7150	7154	6446	6447	6459	4908	4912	4924
1.0	7571	7544	7532	7523	6723	6720	6720	4986	4997	5007
2.0	8286	8073	8003	7960	7024	7002	6984	4962	4996	5021
				$\rho_3=0.30$ (Floor)						
0	—	—	—	—	—	—	—	—	—	—
0.1	0.6626	0.6628	0.6631	0.6632	0.6178	0.6179	0.6180	0.5071	0.5072	0.5073
0.2	6786	6792	6796	6798	6292	6295	6298	5093	5094	5095
0.3	6938	6946	6951	6954	6397	6402	6406	5108	5113	5114
0.4	7078	7087	7094	7101	6492	6500	6505	5118	5123	5129
0.5	7210	7220	7228	7234	6579	6587	6595	5124	5132	5137
0.7	7440	7450	7451	7462	6724	6726	6738	5124	5131	5144
1.0	7721	7710	7705	7705	6872	6877	6883	5100	5116	5132
2.0	8175	8036	7998	7977	6989	6992	7001	4938	4992	5032

TABLE XIII.

Wall Lighting, H_3/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$			k_r
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	
					$\rho_3=0.10$ (Floor)						
0	—	—	—	—	—	—	—	—	—	—	0
0.1	$^{0}1123$	$^{0}1123$	$^{0}1124$	$^{0}1125$	$^{0}1136$	$^{0}1136$	$^{0}1137$	$^{0}1166$	$^{0}1166$	$^{0}1166$	0.1
0.2	1102	1103	1104	1105	1115	1116	1117	1145	1146	1147	0.2
0.3	1081	1085	1086	1088	1096	1099	1100	1126	1128	1130	0.3
0.4	1062	1066	1070	1073	1079	1083	1086	1109	1112	1114	0.4
0.5	1044	1051	1056	1061	1064	1068	1073	1093	1097	1101	0.5
0.7	$^{0}1010$	$^{0}1024$	1032	1041	1037	1044	1053	1064	1070	1077	0.7
1.0	$^{1}9676$	$^{1}9921$	$^{0}1007$	$^{0}1022$	$^{0}1003$	$^{0}1017$	$^{0}1031$	$^{0}1027$	$^{0}1039$	1051	1.0
2.0	$^{1}8664$	$^{1}9288$	$^{1}9631$	$^{1}9932$	$^{1}9351$	$^{1}9685$	$^{1}9964$	$^{1}9472$	$^{1}9782$	$^{0}1004$	2.0
					$\rho_3=0.30$ (Floor)						
0	—	—	—	—	—	—	—	—	—	—	0
0.1	$^{0}3305$	$^{0}3305$	$^{0}3305$	$^{0}3305$	$^{0}3345$	$^{0}3345$	$^{0}3346$	$^{0}3443$	$^{0}3444$	$^{0}3445$	0.1
0.2	3256	3257	3258	3260	3297	3299	3301	3396	3397	3399	0.2
0.3	3209	3213	3217	3221	3255	3258	3263	3351	3354	3358	0.3
0.4	3163	3174	3181	3188	3215	3221	3227	3309	3314	3321	0.4
0.5	3121	3137	3148	3158	3178	3188	3198	3269	3278	3285	0.5
0.7	3043	3074	3091	3111	3113	3128	3147	3198	3209	3225	0.7
1.0	2941	2997	3031	3063	3032	3063	3092	3105	3130	3154	1.0
2.0	2690	2839	2919	2989	2858	2934	2998	2896	2965	3020	2.0

TABLE XIV.

Direct Lighting, f.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$			k_r
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	
					$\rho_3=0.10$ (Floor)						
0	1.0870	1.0870	1.0870	1.0870	1.0753	1.0753	1.0753	1.0526	1.0526	1.0526	0
0.1	1.0806	1.0718	1.0667	1.0619	1.0625	1.0580	1.0538	1.0445	1.0411	1.0380	0.1
0.2	1.0746	1.0596	1.0515	1.0444	1.0522	1.0450	1.0387	1.0380	1.0325	1.0276	0.2
0.3	1.0691	1.0498	1.0401	1.0322	1.0440	1.0353	1.0282	1.0327	1.0260	1.0203	0.3
0.4	1.0642	1.0420	1.0316	1.0235	1.0373	1.0280	1.0207	1.0285	1.0211	1.0151	0.4
0.5	1.0598	1.0357	1.0252	1.0173	1.0320	1.0225	1.0153	1.0251	1.0173	1.0113	0.5
0.7	1.0523	1.0265	1.0167	1.0097	1.0243	1.0152	1.0087	1.0200	1.0123	1.0066	0.7
1.0	1.0436	1.0184	1.0100	1.0045	1.0174	1.0092	1.0041	1.0153	1.0082	1.0034	1.0
2.0	1.0280	1.0091	1.0041	1.0011	1.0090	1.0041	1.0011	1.0089	1.0040	1.0011	2.0
					$\rho_3=0.30$ (Floor)						
0	1.3158	1.3158	1.3158	1.3158	1.2658	1.2658	1.2658	1.1765	1.1765	1.1765	0
0.1	1.2879	1.2509	1.2297	1.2105	1.2138	1.1960	1.1798	1.1465	1.1342	1.1229	0.1
0.2	1.2627	1.2023	1.1711	1.1451	1.1744	1.1477	1.1252	1.1232	1.1040	1.0874	0.2
0.3	1.2408	1.1654	1.1301	1.1023	1.1442	1.1135	1.0840	1.1051	1.0821	1.0632	0.3
0.4	1.2214	1.1371	1.1007	1.0734	1.1208	1.0887	1.0642	1.0908	1.0659	1.0464	0.4
0.5	1.2043	1.1151	1.0793	1.0535	1.1026	1.0706	1.0471	1.0794	1.0538	1.0346	0.5
0.7	1.1761	1.0843	1.0517	1.0296	1.0768	1.0470	1.0264	1.0629	1.0379	1.0201	0.7
1.0	1.1448	1.0578	1.0308	1.0137	1.0544	1.0283	1.0125	1.0478	1.0252	1.0102	1.0
2.0	1.0915	1.0284	1.0126	1.0034	1.0281	1.0125	1.0034	1.0276	1.0123	1.0033	2.0

TABLE XV.

Direct Lighting, H_{1T}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
				$\rho_3=0.10$ (Floor)						
0	17200	14500	12700	28999	14250	12550	28500	13750	12250	27500
0.1	16790	14011	12322	27475	13773	12188	27056	13302	11922	26220
0.2	16378	13556	11989	26204	13334	11867	25854	12898	11638	25156
0.3	15970	13139	11699	25147	12934	11596	24855	12538	11393	24272
0.4	15569	12760	11447	24268	12575	11359	24025	12218	11183	23540
0.5	15181	12421	11231	23538	12255	11155	23335	11936	11004	22932
0.7	14453	11851	28878	22427	11720	28317	22287	11470	27220	22010
1.0	13503	11226	25407	21377	11137	25061	21297	20685	24387	21139
2.0	11490	23017	21017	32070	22794	39544	31949	22373	38265	31711
				$\rho_3=0.30$ (Floor)						
0	02183	01365	18189	12729	01275	17650	12550	01125	16750	12250
0.1	2036	1201	16941	12232	01130	16544	12108	19897	15756	11861
0.2	1913	01063	15935	11849	19975	15589	11746	18689	14901	11541
0.3	1791	19385	15069	11534	18783	14766	11448	17612	14170	11277
0.4	1672	18261	14323	11273	17716	14061	11201	16662	13545	11058
0.5	1557	17253	13681	11056	16764	13456	29959	15820	13012	28769
0.7	1343	15564	12662	27261	15174	12496	26845	14432	12170	26020
1.0	01062	13701	11627	24130	13435	11525	23892	12930	11322	23419
2.0	14588	29206	23091	36227	28526	22892	35866	27243	22506	25148

TABLE XVI.

Direct Lighting, H_{1M}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
				$\rho_3=0.10$ (Floor)						
0	17200	14500	12700	28999	14250	12550	28500	13750	12250	27500
0.1	16872	14066	12356	27592	13848	12234	27213	13420	11993	26456
0.2	16545	13671	12061	26454	13485	11960	26167	13120	11770	25595
0.3	16224	13315	11810	25531	13157	11732	25314	12851	11578	24880
0.4	15908	12996	11595	24774	12855	11533	24609	12607	11411	24281
0.5	15601	12711	11411	24147	12601	11362	24022	12388	11266	23774
0.7	15025	12232	11118	23185	12157	11087	23114	12012	11028	22973
1.0	14262	11693	28085	22222	11651	27941	22191	11571	27653	22131
2.0	12504	27488	23152	37845	27435	23150	37827	27334	23126	37791
				$\rho_3=0.30$ (Floor)						
0	02183	01365	18189	12729	01275	17650	12550	01125	16749	12250
0.1	2060	1217	17041	12266	1153	16682	12155	01025	15967	11931
0.2	1963	01097	16151	11923	01043	15865	11839	19356	15297	11672
0.3	1867	19911	15399	11648	19449	15172	11585	18551	14723	11458
0.4	1774	18965	14762	11423	18578	14582	11375	17830	14227	11280
0.5	1684	18125	14219	11238	17800	14076	11201	17179	13796	11129
0.7	1515	16709	13351	29529	16485	13262	29321	16062	13089	28908
1.0	01292	15112	12434	26664	14988	12392	26574	14753	12306	26396
2.0	17709	12285	29581	22361	12269	29547	22356	12239	29474	22345

TABLE XVII.

Direct Lighting, H_2/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	18000	18000	18000	18000	17000	17000	17000	15000	15000	15000
0.1	17544	17132	16880	16645	16214	16006	15811	14402	14270	14146
0.2	17086	16322	15893	15515	15490	15136	14821	13864	13639	13437
0.3	16633	15580	15033	14576	14832	14380	13998	13383	13096	12848
0.4	16188	14907	14289	13794	14242	13729	13314	12958	12630	12359
0.5	15757	14303	13648	13145	13714	13170	12746	12581	12232	11954
0.7	14948	13291	12631	12158	12833	12283	11884	11960	11605	11340
1.0	13893	12179	11602	11224	11872	11389	11068	11291	20749	27593
2.0	11655	25363	23023	21839	24602	22619	21605	23164	21837	21140
					$\rho_3=0.30$ (Floor)					
0	02426	02426	02426	02426	02100	02100	02100	01500	01500	01500
0.1	2262	2134	2056	1984	1861	1796	1736	1320	1279	1241
0.2	2125	1890	1759	1643	1643	1534	1438	1158	01089	01027
0.3	1990	1668	1502	1363	1447	1309	01192	01015	19269	18513
0.4	1858	1469	1281	01131	1271	01115	19892	18881	17877	17055
0.5	1731	01290	01091	19383	01114	19486	18203	17760	16692	15846
0.7	1492	19888	17888	16454	18522	16851	15638	15910	14821	14014
1.0	01180	16579	14821	13671	15658	14185	13205	13907	12937	12280
2.0	15097	11637	20157	25535	11404	27938	24830	29657	25565	23432

TABLE XVIII.

Direct Lighting, H_3/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	01000	01000	01000	01000	01000	01000	01000	01000	01000	01000
0.1	19997	19988	19980	19976	19990	19983	19979	19997	19991	19985
0.2	19997	19982	19973	19965	19987	19977	19971	19997	19988	19980
0.3	19999	19982	19974	19965	19988	19978	19969	01000	19989	19980
0.4	01000	19986	19977	19968	19993	19983	19973	1000	19993	19983
0.5	1001	19993	19983	19974	01000	19988	19978	1001	19998	19987
0.7	1002	01001	19997	19986	1001	01000	19988	1002	01001	19995
1.0	1005	1003	01002	01000	1004	1002	01000	1004	1002	01000
2.0	1013	1009	1005	1002	1009	1005	1002	1009	1005	1002
					$\rho_3=0.30$ (Floor)					
0	03000	03000	03000	03000	03000	03000	03000	03000	03000	03000
0.1	2997	2989	2983	2978	2992	2986	2980	2996	2991	2986
0.2	2998	2984	2976	2969	2988	2980	2973	2997	2989	2982
0.3	3000	2985	2976	2968	2990	2981	2973	3000	2990	2982
0.4	3004	2988	2979	2972	2994	2984	2976	3004	2993	2983
0.5	3009	2994	2985	2976	2999	2989	2980	3009	2998	2987
0.7	3022	3008	2997	2987	3012	3001	2989	3021	3007	2994
1.0	3045	3029	3014	2999	3032	3018	3000	3038	3020	3002
2.0	3119	3077	3044	3014	3078	3045	3014	3079	3045	3014

TABLE XIX.

Wall and Ceiling Lighting, f.

k_r	$\rho_1=0.80$ (Ceiling)				$\rho_1=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	0 8695	0 8695	0 8695	0 8695	0 7527	0 7527	0 7527	0 5263	0 5263	0 5263
0.1	7875	6771	6104	5488	5919	5327	4776	4281	3820	3387
0.2	7313	5574	4619	3790	4917	4063	3315	3657	2982	2381
0.3	6869	4730	3653	2764	4208	3236	2428	3207	2427	1766
0.4	6490	4097	2975	2091	3671	2655	1845	2860	2033	1359
0.5	6156	3598	2477	1626	3247	2226	0 1441	2581	1739	0 1075
0.7	5576	2861	1801	0 1043	2618	1642	1 9329	2158	0 1331	1 7142
1.0	4864	2145	0 1219	1 5970	2012	0 1132	1 5420	1724	1 9644	1 4328
2.0	0 3323	0 1112	1 5281	1 1735	1081	1 5136	1 1661	0 1023	1 4854	1 1514
					$\rho_3=0.30$ (Floor)					
0	1.0526	1.0526	1.0526	1.0526	0 8861	0 8861	0 8861	0 5883	0 5883	0 5883
0.1	0 9399	0 7892	0 7016	0 6228	6758	6009	5326	4702	4158	3655
0.2	8600	6274	5122	4128	5480	4446	3571	3958	3183	2511
0.3	7969	5237	3949	2932	4603	3467	2556	3447	2555	1835
0.4	7438	4455	3159	2180	3957	2801	1913	3032	2118	1397
0.5	6977	3859	2595	1674	3460	2325	0 1479	2716	1798	0 1097
0.7	6203	3010	1856	1059	2744	1687	1 9458	2245	0 1362	1 7237
1.0	5296	2220	0 1240	1 6010	2066	0 1151	1 5454	1777	1 9792	1 4368
2.0	0 3496	0 1131	1 5320	1 1738	0 1099	1 5177	1 1664	0 1039	1 4892	1 1517

TABLE XX.

Wall and Ceiling Lighting, H_{1T}/D_{AV}.

k_r	$\rho_1=0.80$ (Ceiling)				$\rho_1=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	1.3600	0 8500	0 5100	0 1700	0 9393	0 5636	0 1879	1.2251	0 7350	0 2450
0.1	1.2322	8421	5388	1924	9184	5901	2118	1.1506	7495	2732
0.2	1.1643	8475	5710	2169	9140	6204	2379	1.1080	7689	3036
0.3	1.1267	8615	6060	2439	9193	6532	2666	1.0832	7913	3364
0.4	1.1061	8793	6428	2735	9301	6877	2977	1.0688	8153	3713
0.5	1.0955	9003	6810	3057	9443	7232	3314	1.0608	8399	4083
0.7	1.0915	0 9467	7592	3779	0 9775	7948	4062	1.0551	8883	4878
1.0	1.1034	1.0163	0 8751	5046	1.0285	0 8982	5342	1.0572	0 9549	6151
2.0	1.1526	1.1749	1.1578	0 9716	1.1384	1.1346	0 9752	1.0632	1.0852	0 9841
					$\rho_3=0.30$ (Floor)					
0	1.2800	0 8000	0 4800	0 1600	0 8892	0 5336	0 1779	1.1749	0 7050	0 2350
0.1	1.1802	8075	5171	1847	8840	5686	2042	1.1169	7281	2656
0.2	1.1277	8272	5548	2112	8891	6043	2322	1.0840	7532	2978
0.3	1.0986	8418	5935	2394	9002	6409	2621	1.0598	7792	3317
0.4	1.0826	8638	6329	2699	9146	6779	2942	1.0535	8055	3703
0.5	1.0748	8871	6728	3029	9310	7138	3286	1.0474	8317	4051
0.7	1.0720	0 9353	7528	3761	0 9660	7882	4043	1.0431	8817	4848
1.0	1.0818	1.0048	0 8694	5033	1.0168	0 8923	5328	1.0450	0 9486	6114
2.0	1.1231	1.1612	1.1510	9701	1.1248	1.1275	9737	1.0503	1.0786	9824

TABLE XXI.

Wall and Ceiling Lighting, H_{1M}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	1.3600	0.8500	0.5100	0.1700	0.9393	0.5636	0.1879	1.2251	0.7349	0.2450
0.1	1.2032	8215	5254	1875	8984	5770	2069	1.1319	7368	2684
0.2	1.1152	8094	5443	2064	8775	5944	2274	1.0763	7450	2935
0.3	1.0622	8075	5659	2269	8686	6147	2498	1.0420	7573	3203
0.4	1.0294	8109	5891	2491	8671	6367	2738	1.0203	7721	3488
0.5	1.0087	8184	6138	2731	8701	6601	2996	1.0069	7884	3790
0.7	0.9888	8404	6648	3264	8842	7080	3564	0.9945	8220	4432
1.0	0.9862	8795	7412	4183	9135	7788	4523	0.9933	8712	5453
2.0	1.0273	9802	9256	7459	9914	9415	7731	1.0141	9741	8354
					$\rho_3=0.30$ (Floor)					
0	1.2799	0.8000	0.4800	0.1600	0.8893	0.5335	0.1779	1.1749	0.7050	0.2350
0.1	1.1576	7865	5060	1807	8686	5578	2001	1.1025	7180	2616
0.2	1.0894	7927	5322	2019	8567	5824	2231	1.0568	7334	2890
0.3	1.0483	7966	5585	2241	8584	6075	2469	1.0279	7506	3174
0.4	1.0228	8054	5851	2474	8617	6328	2721	1.0159	7685	3470
0.5	1.0071	8164	6119	2722	8681	6573	2988	1.0054	7868	3777
0.7	0.9911	8415	6652	3265	8854	7086	3565	0.9957	8226	4424
1.0	0.9891	8813	7425	4188	9151	7800	4528	0.9948	8722	5439
2.0	1.0218	9778	9245	7457	9888	9395	7730	1.0115	9727	8351

TABLE XXII.

Wall and Ceiling Lighting, H_2/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9660	1.0281	1.0764	1.1314	1.0157	1.0679	1.1284	0.9778	1.0422	1.1182
0.2	9561	1.0638	1.1575	1.2768	1.0401	1.1399	1.2694	9709	1.0874	1.2464
0.3	9578	1.1044	1.2424	1.4375	1.0692	1.2148	1.4245	9717	1.1343	1.3844
0.4	9656	1.1460	1.3300	1.6138	1.1006	1.2913	1.5934	9768	1.1812	1.5320
0.5	0.9766	1.1890	1.4196	1.8059	1.1327	1.3677	1.7764	0.9839	1.2276	1.6885
0.7	1.0030	1.2741	1.6001	2.238	1.1961	1.5215	2.813	1.0002	1.3151	2.024
1.0	1.0431	1.3920	1.8634	2.997	1.2820	1.7383	2.880	1.0229	1.4516	2.561
2.0	1.1313	1.6445	2.494	5.788	1.4513	2.224	5.276	1.0549	1.6484	4.113
					$\rho_3=0.30$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9736	1.0332	1.0795	1.1324	1.0208	1.0712	1.1307	0.9828	1.0453	1.1193
0.2	9659	1.0770	1.1619	1.2789	1.0470	1.1444	1.2711	9774	1.0918	1.2478
0.3	9672	1.1108	1.2470	1.4393	1.0759	1.2194	1.4264	9735	1.1386	1.3855
0.4	9731	1.1518	1.3341	1.6153	1.1063	1.2953	1.5950	9819	1.1849	1.5326
0.5	0.9817	1.1931	1.4226	1.8074	1.1367	1.3728	1.7777	0.9874	1.2302	1.6876
0.7	1.0024	1.2740	1.6003	2.239	1.1961	1.5217	2.184	1.0002	1.3153	2.020
0.1	1.0341	1.3857	1.8591	2.995	1.2759	1.7340	2.878	1.0180	1.4270	2.550
2.0	1.1058	1.6273	2.481	5.781	1.4358	2.292	5.269	1.0437	1.6393	4.107

TABLE XXIII.

Wall and Ceiling Lighting, H_3/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	01000	01000	01000	01000	01000	01000	01000	01000	01000	01000
0.1	01009	19904	19763	19604	19943	19791	19614	01006	19867	19643
0.2	01007	19789	19548	19238	19854	19597	19259	01004	19739	19321
0.3	19991	19673	19356	18910	19754	19421	18947	19999	19624	19040
0.4	19895	19551	19188	18615	19654	19274	18668	19939	19523	18796
0.5	19786	19441	19045	18355	19560	19147	18425	19874	19438	18590
0.7	19559	19252	18828	17937	19390	18959	18049	19735	19307	18289
1.0	19234	19050	18666	17579	19200	18824	17748	19555	19214	18090
2.0	18406	18801	18851	18092	18916	18982	18325	19147	19252	18707
					$\rho_3=0.30$ (Floor)					
0	03000	03000	03000	03000	03000	03000	03000	03000	03000	03000
0.1	3020	2968	2927	2880	2980	2934	2883	3013	2958	2892
0.2	3016	2934	2861	2771	2953	2876	2777	3010	2920	2795
0.3	2999	2899	2806	2672	2926	2826	2682	2999	2886	2710
0.4	2975	2869	2757	2584	2900	2784	2598	2985	2858	2637
0.5	2949	2840	2717	2507	2876	2744	2525	2969	2835	2575
0.7	2896	2792	2657	2385	2834	2697	2411	2938	2801	2484
1.0	2818	2743	2616	2279	2788	2663	2319	2895	2781	2423
2.0	2614	2693	2684	2436	2727	2723	2492	2796	2805	2621

TABLE XXIV.

Uniformly Diffusing Globes, f.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	09783	09783	09783	09783	09140	09140	09140	07895	07895	07895
0.1	9015	8328	7915	7532	7754	7388	7048	6650	6367	6102
0.2	8407	7217	6563	6264	6709	6129	5620	5737	5284	4882
0.3	7895	6330	5542	5051	5885	5180	4593	5033	4477	4004
0.4	7445	5606	4744	4161	5215	4441	3823	4470	3852	3348
0.5	7042	5000	4104	3486	4657	3850	3229	4007	3356	2842
0.7	6334	4049	3156	2543	3788	2975	2385	3293	2622	2122
1.0	5460	3057	02240	01700	2884	02129	01615	2556	01913	01463
2.0	03572	01513	19762	16542	01467	19519	16391	01379	19210	16156
					$\rho_3=0.30$ (Floor)					
0	1.1842	1.1842	1.1842	1.1842	1.0759	1.0759	1.0759	08824	08824	08824
0.1	1.0750	09712	09111	08568	08854	08343	07877	7300	6947	6595
0.2	09877	8147	7292	6553	7480	6718	6073	6207	5681	5158
0.3	9151	7011	6003	5206	6439	5558	4851	5397	4775	4167
0.4	8525	6099	5044	4249	5622	4691	3976	4736	4094	3447
0.5	7973	5364	4307	3536	4963	4023	3323	4215	3563	2905
0.7	7041	4260	3255	2561	3970	3061	2422	3426	2795	2150
1.0	5944	3165	02281	01705	2980	02166	01627	2633	2065	01475
2.0	03755	01538	19836	16545	01491	19594	16405	01402	01025	16125

TABLE XXV.

Uniformly Diffusing Globes, H_{1T}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	$^{0}5955$	$^{0}3723$	$^{0}2233$	$^{1}7444$	$^{0}3632$	$^{0}2179$	$^{1}7265$	$^{0}3416$	$^{0}2050$	$^{1}6833$
0.1	6682	4250	2580	$^{1}8702$	4335	2631	$^{1}8872$	4537	2753	$^{1}9287$
0.2	6944	4491	2757	$^{1}9007$	4566	2803	$^{1}9568$	4743	2914	$^{1}9947$
0.3	7216	4744	2946	$^{1}9853$	4804	2985	$^{0}1031$	4951	3079	$^{0}1065$
0.4	7492	5007	3146	$^{0}1075$	5051	3176	1111	5157	3249	1139
0.5	7768	5278	3358	1168	5304	3377	1197	5366	3423	1217
0.7	8309	5841	3805	1369	5821	3797	1383	5772	3780	1381
1.0	$^{0}9075$	6712	4532	1703	6603	4469	1693	6361	4327	1647
2.0	1.1173	9490	7065	2969	9014	6735	2844	8031	5940	2562
					$\rho_3=0.30$ (Floor)					
0	$^{0}3511$	$^{0}2195$	$^{0}1317$	$^{1}4389$	$^{0}2088$	$^{0}1253$	$^{1}4177$	$^{0}1833$	$^{0}1100$	$^{1}3667$
0.1	7074	4498	2729	$^{1}9199$	4593	2787	$^{1}9393$	4822	2919	$^{1}9864$
0.2	7281	4721	2887	$^{1}9850$	4786	2938	$^{0}1002$	4983	3041	$^{0}1044$
0.3	7492	4924	3058	$^{0}1056$	4990	3100	1071	5136	3162	1107
0.4	7711	5155	3241	1133	5203	3273	1146	5319	3287	1174
0.5	7934	5399	3436	1218	5426	3454	1226	5490	3416	1246
0.7	8374	5906	3855	1404	5887	3847	1403	5841	3680	1401
1.0	$^{0}9021$	6711	4548	1723	6606	4486	1704	6363	4086	1655
2.0	1.0895	9388	7028	2972	8911	6701	2841	7944	5349	2567

TABLE XXVI.

Uniformly Diffusing Globes, H_{1M}/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	$^{0}5955$	$^{0}3723$	$^{0}2233$	$^{1}7444$	$^{0}3632$	$^{0}2179$	$^{1}7265$	$^{0}3416$	$^{0}2050$	$^{1}6833$
0.1	6525	4146	2515	$^{1}8480$	4240	2572	$^{1}8671$	4468	2709	$^{1}9131$
0.2	6631	4273	2618	$^{1}8534$	4370	2677	$^{1}9116$	4600	3819	$^{1}9599$
0.3	6750	4406	2723	$^{1}9062$	4501	2783	$^{1}9569$	4733	2927	$^{0}1007$
0.4	6880	4542	2830	$^{1}9589$	4637	2891	$^{0}1003$	4866	3037	1055
0.5	7012	4680	2941	$^{0}1011$	4773	3002	1051	4996	3146	1103
0.7	7286	4959	3164	1115	5048	3223	1148	5254	3362	1201
1.0	7694	5377	3504	1272	5451	3556	1300	5622	3677	1348
2.0	8816	6528	4494	1760	6547	4513	1773	6585	4473	1780
					$\rho_3=0.30$ (Floor)					
0	$^{0}3511$	$^{0}2195$	$^{0}1317$	$^{1}4389$	$^{0}2088$	$^{0}1253$	$^{1}4177$	$^{0}1833$	$^{0}1100$	$^{1}3667$
0.1	6959	4420	2680	$^{1}9030$	4527	2744	$^{1}9246$	4784	2893	$^{1}9766$
0.2	7050	4551	2777	$^{1}9451$	4642	2842	$^{1}9667$	4898	2979	$^{0}1020$
0.3	7143	4656	2874	$^{1}9869$	4759	2939	$^{0}1009$	4997	3057	1063
0.4	7247	4775	2972	$^{0}1029$	4877	3036	1052	5120	3131	1107
0.5	7355	4897	3072	1074	4997	3135	1096	5232	3204	1151
0.7	7569	5143	3276	1165	5234	3337	1188	5449	3343	1245
1.0	7899	5510	3587	1308	5588	3641	1329	5762	3543	1377
2.0	8831	6557	4517	1774	6575	4535	1783	6616	4070	1804

TABLE XXVII.

Uniformly Diffusing Globes, H_2/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	$^{0}8889$	$^{0}8889$	$^{0}8889$	$^{0}8889$	$^{0}8236$	$^{0}8236$	$^{0}8236$	$^{0}6668$	$^{0}6668$	$^{0}6668$
0.1	5653	5636	5625	5613	5230	5218	5207	4247	4236	4226
0.2	6309	6342	6366	6117	5885	5905	5929	4776	4796	4812
0.3	6887	7025	7120	6993	6508	6598	6693	5271	5347	5430
0.4	7413	7692	7892	7921	7114	7305	7512	5738	5898	6072
0.5	7897	8352	$^{0}8694$	$^{0}8907$	7711	8029	$^{0}8377$	6187	6454	6749
0.7	8775	$^{0}9669$	1.0361	1.1038	$^{0}8881$	$^{0}9526$	1.0264	7039	7574	$^{0}8188$
1.0	$^{0}9932$	1.1655	1.3054	1.4649	1.0607	1.1898	1.3453	$^{0}8236$	$^{0}9285$	1.0549
2.0	1.2917	1.8067	2.278	2.915	1.5988	2.025	2.604	1.1689	1.4668	1.9208
					$\rho_3=0.30$ (Floor)					
0	$^{0}5778$	$^{0}5778$	$^{0}5778$	$^{0}5778$	$^{0}5353$	$^{0}5353$	$^{0}5353$	$^{0}4333$	$^{0}4333$	$^{0}4333$
0.1	6374	6338	6332	6313	5892	5874	5858	4788	4765	4758
0.2	6891	6939	6932	6949	6417	6432	6448	5215	5195	5240
0.3	7350	7481	7575	7672	6936	7022	7621	5607	5625	5772
0.4	7772	8052	8259	8477	7453	7644	8491	6016	6060	6353
0.5	8168	8635	$^{0}8981$	$^{0}9361$	7973	8295	$^{0}9416$	6401	6502	6974
0.7	8894	$^{0}9824$	1.0538	1.1351	$^{0}9020$	$^{0}9686$	1.1437	7154	7403	$^{0}8332$
1.0	$^{0}9881$	1.1665	1.3104	1.4825	1.0617	1.1953	1.4849	$^{0}8245$	$^{0}8775$	1.0608
2.0	1.2583	1.7864	2.305	2.951	1.5806	2.014	2.849	1.1555	1.3207	1.9327

TABLE XXVIII.

Uniformly Diffusing Globes, H_3/D_{AV}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$	$\rho_1=0.50$	$\rho_1=0.30$	$\rho_1=0.10$
					$\rho_3=0.10$ (Floor)					
0	$^{0}6002$	$^{0}6002$	$^{0}6002$	$^{0}6002$	$^{0}6294$	$^{0}6294$	$^{0}6294$	$^{0}6999$	$^{0}6999$	$^{0}6999$
0.1	$^{1}1003$	$^{1}9934$	$^{1}9867$	$^{1}9801$	$^{1}9956$	$^{1}9886$	$^{1}9817$	$^{1}1001$	$^{1}9934$	$^{1}9861$
0.2	$^{1}9994$	$^{1}9838$	$^{1}9726$	$^{1}9200$	$^{1}9878$	$^{1}9762$	$^{1}9644$	$^{1}9976$	$^{1}9854$	$^{1}9726$
0.3	$^{1}9919$	$^{1}9728$	$^{1}9585$	$^{1}9137$	$^{1}9784$	$^{1}9639$	$^{1}9484$	$^{1}9921$	$^{1}9768$	$^{1}9603$
0.4	$^{1}9824$	$^{1}9615$	$^{1}9452$	$^{1}9063$	$^{1}9684$	$^{1}9520$	$^{1}9341$	$^{1}9857$	$^{1}9686$	$^{1}9492$
0.5	$^{1}9717$	$^{1}9506$	$^{1}9332$	$^{1}8982$	$^{1}9590$	$^{1}9416$	$^{1}9216$	$^{1}9793$	$^{1}9607$	$^{1}9395$
0.7	$^{1}9500$	$^{1}9304$	$^{1}9135$	$^{1}8836$	$^{1}9409$	$^{1}9234$	$^{1}9019$	$^{1}9657$	$^{1}9474$	$^{1}9250$
1.0	$^{1}9176$	$^{1}9078$	$^{1}8946$	$^{1}8706$	$^{1}9202$	$^{1}9066$	$^{1}8869$	$^{1}9487$	$^{1}9348$	$^{1}9142$
2.0	$^{1}8379$	$^{1}8806$	$^{1}8976$	$^{1}8993$	$^{1}8931$	$^{1}9094$	$^{1}9119$	$^{1}9188$	$^{1}9179$	$^{1}9292$
					$\rho_3=0.30$ (Floor)					
0	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$	$^{0}3000$
0.1	3006	2975	2954	2932	2983	2961	2938	3000	2970	2952
0.2	2998	2946	2910	2873	2960	2922	2883	2991	2933	2910
0.3	2980	2915	2870	2820	2934	2886	2836	2978	2891	2873
0.4	2957	2886	2833	2774	2909	2854	2795	2963	2850	2841
0.5	2932	2858	2800	2736	2884	2824	2758	2947	2811	2813
0.7	2879	2808	2747	2676	2840	2778	2705	2915	2738	2772
1.0	2802	2750	2698	2627	2789	2735	2664	2875	2654	2742
2.0	2604	2695	2722	2711	2731	2758	2745	2810	2519	2816

Sonar Uses Doppler Effect.—(*Electronics*, Vol. 19, No. 10.) A well-known scientific principle, the Doppler effect, provides the means for determining the motion and speed of a moving under-water target with echo ranging equipment. If the distance between the sonar-equipped search vessel and the target is increasing, the frequency of the echo will be lower than that of the outgoing pulse. Conversely, if the distance is decreasing, the echo frequency will be higher.

By comparing the frequencies of the echo and transmitted pulse, the sonar operator can determine the relative motion of the target with respect to the ship. This is less important, however, than determination of the absolute motion of the target. Water discontinuities, such as surface waves, provide fixed reflecting points (reverberations) that provide a more useful reference than the outgoing pulse does. By comparing the frequency of the target echo with the frequency of the reverberations, the operator can determine if the target is moving and whether it is moving toward or away from the projector. The change in frequency is 17 cycles per second per knot of absolute speed of the target when echo ranging at a frequency of 24 kc. Sonar operators are trained to distinguish frequency changes as small as 10 cycles, hence target speed can be determined to an accuracy of better than 1 knot.

R. H. OPPERMANN.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING.

The stated monthly meeting of The Franklin Institute was held at 8:15 on Wednesday, November 20, 1946, in the Lecture Hall. Mr. Richard T. Nalle, our newly elected president, was the presiding officer. The President called the meeting to order and announced that the minutes of the October meeting were published in full in the November issue of the JOURNAL, and that, unless he heard anything to the contrary, the minutes would be declared approved as printed. There was no dissent.

Mr. Nalle then called upon Dr. H. B. Allen, Secretary, who gave the following membership report:

The elections for the month of October were:

Active	56
Associate	21
Student	29
	106
Total membership as of October 31, 1946	5,533

Mr. Nalle then introduced the speaker of the evening, Dr. Ellice McDonald, Director of the Biochemical Research Foundation, who gave his annual report on the work of the Foundation. Dr. McDonald stated that there were still certain restrictions on the work done at the Foundation, but his report did cover some very interesting experiments on the effects of neutrons on animals. He reported work on studies of enzymes, blood, electrophoretic work and other phases. Dr. McDonald said that most of the year had been concerned with problems for the Manhattan Engineering District.

The meeting was adjourned at 9:45 after the audience had been given the opportunity to question Dr. McDonald.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9×14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

GLIDDEN, H. K., H. F. LOWES AND J. E. COWLES. Airports: Design, Construction and Management. 1946.

MCFARLAND, ROSS A. Human Factors in Air Transport Design. 1946.

ARCHITECTURE AND BUILDING.

GRISWOLD, JOHN. Fuels, Combustion and Furnaces. 1946.

STEINER, KALMAN. Fuels and Fuel Burners. 1946.

ASTRONOMY.

BARTON, W. H., AND J. M. JOSEPH. Starcraft. 1946.
KOPAL, ZDENEK. An Introduction to the Study of Eclipsing Variables. 1946.

BIOGRAPHIES.

BATHE, GREVILLE. Citizen Genet; Diplomat and Inventor. 1946.

BOTANY.

SINNOTT, EDMUND W. Botany, Principles and Problems. 1946.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

Chemical Engineering Catalog. 31st Edition. 1946.
LUDER, W. F., AND S. ZUFFANTI. The Electronic Theory of Acids and Bases. 1946.
NIGGLI, PAUL. Grundlogen der Stereochemie. 1945.
Organic Reactions. Volume 3. 1946.
PETERS, J. P., AND D. VAN SLYKE. Quantitative Clinical Chemistry. Volume 1. 1946.
PILCHER, R. B., AND BUTLER-JONES. What Industry Owes to Chemical Science. 1946.
ROCHOW, EUGENE G. An Introduction to the Chemistry of the Silicones. 1946.
THEILHEIMER, W. Synthetische Methoden der Organischen Chemie. Volume 1. 1946.
THOMPSON, HAROLD W. Course in Chemical Spectroscopy. 1938.

DICTIONARIES.

DEVRIES, LOUIS. German-English Science Dictionary. 1946.

ELECTRICAL ENGINEERING.

ATKINSON, A. D. S. Fluorescent lighting. 1946.
LOEB, LEONARD, AND JOHN MEEK. The Mechanism of the Electric Spark. 1941.
SARBACHER, R. I., AND W. A. EDSON. Hyper and Ultrahigh Frequency Engineering. 1946.
SELGIN, PAUL J. Electrical Transmission in Steady State. 1946.
STILL, ALFRED. Soul of Lodestone; the Background of Magnetical Science. 1946.
YOUNG, VICTOR JAY. Understanding Microwaves. 1946.

ENGINEERING.

MCCULLOUGH J., AND C. MCCULLOUGH. The Engineer at Law. Volumes 1 and 2. 1946.
MILLS, JOHN. The Engineer in Society. 1946.

INDUSTRIAL MANAGEMENT.

GRANT, EUGENE L. Statistical Quality Control. 1946.

MATHEMATICS.

BELL, ERIC TEMPLE. The Magic of Numbers. 1946.
CHURCHILL, RUEL VANCE. Fourier Series and Boundary Value Problems. 1941.
CRAMER, HARALD. Mathematical Methods of Statistics. 1946.
DULL, RAYMOND W. Mathematical Aids for Engineers. 1946.
PURDON, F. F., AND V. W. SLATER. Aqueous Solution and the Phase Diagram. 1946.
U. S. Mathematical Tables Project. Tables of Fractional Powers. 1946.
VOLTERRA, VITO. Theory of Functions and of Integral and Integro-differentials. 1931.

METALLURGY.

PERAULT, ROBERT. Le Cobalt. 1946.

OPTICS.

HAWKINS, GEORGE A. Thermodynamics. 1946.
LUCKIESH, MATTHEW. Applications of Germicidal, Erythmal and Infrared Energy. 1946.

PETROLEUM.

UREN, LESTER CHARLES. Petroleum Production Engineering, Oil Field Development. 1946.

PHARMACY AND PUBLIC HEALTH.

POWERS, WENDELL H. Chemotherapy. 1946.

PHYSICS.

GREENEWALT, MARY HALLOCK. Nourathar; the Fine Art of Light Color Playing. 1946.
NEWTON, SIR ISAAC. Sir Isaac Newton: Mathematical Principles. 1946.

SANITARY ENGINEERING.

DICKEY, G. D., AND C. L. BRYDEN. Theory and Practice of Filtration. 1946.

SCIENCE.

Mexican-American Conference on Industrial Research Proceedings. 1946.

STATISTICS.

Columbia University Statistical Research Group. Sequential Analysis of Statistical Data. 1945.

SUGAR.

LYLE, OLIVER. Technology for Sugar Refinery Workers. 1941.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

An index is now available to the *Notes from the Biochemical Research Foundation* from Nov. 1938 to Dec. 1945 and to BRF Publications Nos. 1 to 212 which have appeared in various journals from 1929 to 1945. These *Notes* and Publications have been bound biannually in *Reports of the Biochemical Research Foundation of the Franklin Institute*, Vols. I to VIII, 1930–1945, but not until now has there been an index to them. The index has been prepared by the librarian, Mrs. Anne Himmelberger Longenbach, and may be obtained upon request to the Biochemical Research Foundation, Newark, Delaware.

Abstract of **Physical and Chemical Studies on Southern Bean Mosaic Virus. II. Crystallization by Dialysis.**—GAIL LORENZ MILLER AND W. C. PRICE (*Archives of Biochemistry*, **11**: 329, 1946). Crystallization of southern bean mosaic virus to yield various modifications of rhombic plates and prisms was observed during dialysis at room temperature against media of distilled water, tap water and acetate buffer at pH 5.5. The pH of the dialysis medium and the purity of the starting material were found to be important to the crystallization process. Yields of crystals as high as 89 per cent. were obtained, and recrystallization was readily carried out. The method, in contrast to salting-out procedures, yielded a product which was at the same time crystalline and of low ash content. A tendency of the crystals to dissolve when placed in contact with glass and their glutinous character when mechanically pressed against the inner surfaces of their containers were noted. The identity of the crystals with the virus protein itself was established.

Abstract of **Physical and Chemical Studies on Southern Bean Mosaic Virus. III. Electrophoretic and Nucleic Acid Studies.**—GAIL LORENZ MILLER AND W. C. PRICE (*Archives of Biochemistry*, **11**: 337, 1946). Electrophoretic studies on southern bean mosaic virus revealed a single component possessing an electrochemical stability range at 0° C. of pH 2.92–11.50 and an isoelectric point of pH 5.50. Outside the stability range, at pH 1.97 and 12.45, the virus was split into multiple components with abnormal mobilities. Sedimentation studies also revealed that the particles in the acid solution were split into components with sedimentation constants of the order of 59 S and 10 S, as contrasted with a sedimentation constant of 114 S for the intact virus.

Nucleic acid studies demonstrated that the phosphorus, and, therefore, by inference, the nucleic acid, of the virus was completely set free

after treatment for 60 minutes with 5 per cent. NaOH at room temperature. The simultaneous cleavage of nitrogen, also measured, indicated the formation of other soluble nitrogen-containing fractions besides nucleic acid. Heat denaturation of southern bean mosaic virus did not free the nucleic acid from the protein portion of the virus particle. The phosphorus content of the virus was calculated to correspond to the presence of 21 per cent. nucleic acid.

BOOK REVIEWS.

SOUL OF LODESTONE, by Alfred Still. 233 pages, 14 × 21 cms. New York, Murray Hill Cooks, Inc. 1946. Price $2.50.

This book is a history of man's thoughts concerning the property magnetism which is called the "soul" of lodestone. This mysterious force has been the object of speculation and experiment for centuries. Toward the end of the fourth century the poet Claudian wrote: "There is a stone, colorless and without brilliance, . . . but it is preferred to all the most precious products of the Orient by those who know its virtues and its wonders. Iron gives it life; iron recognizes and nourishes it; when the iron is withdrawn it experiences the torments of hunger and thirst; it dies. . . . Iron and the loadstone are drawn together and united. What can be this subtil flame which, entering these two metals, can give rise to this sympathy? What is the unknown charm which can unite them with a common will and a single desire?"

The questions were left unanswered; they remain unanswered today. The note of wonder endured for many centuries. When it became the fashion among educated men to investigate the workings of nature, the thinking philosophers invented many ingenious hypotheses. These, although far from perfect, at least gave pleasure to their sponsors. They were examined and studied by contemporary and later philosophers; but, in the end, they were invariably abandoned. No theory has ever survived the test of time; it has never penetrated deeply enough into the mysterious phenomena of nature.

The story begins in ancient times, and progresses through the dark ages and the decline of science, the origin of the magnetic compass, William Gilbert and his era, cosmic theories, megnetism after Gilbert, systems and conjectures, and a treatment on modern theories and investigations.

R. H. OPPERMANN.

THEORY AND PRACTICE OF FILTRATION, by George D. Dickey and Charles L. Bryden. 346 pages, drawings and illustrations, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $6.00.

The subject of filtration has long been considered as of only casual importance to the lay mind and rarely of great interest even to the scientist; yet in its broadest sense filtration is one of the most comprehensive of all physical operations and it has had a vital part in man's development throughout his history. The book at hand gives a general concept of filtration as applied to the separation of solids from liquids and gases by the use of porous media. In doing so gas filtration is only briefly touched upon, stress being placed on the separation of solids from liquids.

Extensive literature is lacking on the history of filtration, so that the brief discussion in the beginning of the book is quite in order and also for giving a background of understanding of its primary function. In beginning a discussion on the theory and principles of filtration it is pointed out that it is not the simple process where application of rule of thumb is all that is necessary, but that in reality it is one of the more difficult steps in chemical engineering, and therefore the development of principles and details is given in considerable detail. Reduced to basic terms it is shown that filtration has three factors, all variable for a given slurry at a constant temperature: namely, filter medium, filter cake, and pressure differential; the object is to obtain the greatest capacity of the desired product in the shortest time or upon the smallest filter area (in a continuous-flow setup). This section of the book may be classed as the first part.

The major portion of the book deals with filters and auxiliary apparatus including typical data. There are chapters on gravity filters, pressure filters, and vacuum filters. Of particular interest is a chapter on hydraulic presses and squeeze presses, covering a wide range. Labora-

tory filters are given space, for analytical equipment or miniature plant units. Special mention is made of Oil Filters, extractors and expellers; water filtration; and sewage clarification and sludge dewatering. Aside from these chapters and the appendex chapter on Surface and Interfacial Separation of Immiscible Fluids, no particular application is considered separately but only as a part of the whole, to illustrate the functioning of different types of filtration apparatus. The latter chapters deal with centrifugals and centrifuges; air, gas, and light filters; auxiliary equipment; flow sheets; testing and selection of equipment; installation and operation. In the back there is a subject index.

The treatment is quite comprehensive within its stated limitations. It is an up-to-date work which should be useful in many diversified activities.

R. H. OPPERMANN.

WEISS MAGNETONS AS COMPONENTS OF NUCLEAR AND SUBNUCLEAR STRUCTURES, by Theodore Van Schelven. 32 pages, drawings, 16 × 24 cms. Amsterdam (Holland) Kosmos Publishing Company, 1945. Price $3.00 (paper).

In this pamphlet the author discusses the Magneton theory as a solution of one of the outstanding puzzles of physics—the composition of the atomic nucleus. In 1911, from the then available results for atomic moments, Pierre Weiss concluded that there was a fundamental unit magneton. From later results, particularly on solutions, Weiss concluded in 1924 that the empirical unit magneton has a moment very approximately one-fifth of the Bohr theoretical unit magneton or electron: $M_B = 4.967\ M_W$. Since the Weiss magneton, forming an indispensable unit in terms of which to express deduced atomic moments, constitutes the unique empirical element smaller than electron and positron, and also the smallest single magnetic element experiment ever showed, this unit may be conceived as component of the nucleus.

In the back there are drawings of nuclei of different elements including uranium. Reference is made to this in discussion of what happens in the atomic bomb explosion, in the light of the magneton theory.

R. H. OPPERMANN.

ADVANCING FRONT IN CHEMISTRY, edited by Wendell H. Powers. Volume II. 156 pages, illustrations, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1945. Price $3.25.

This Volume II of a symposium as part of a course of academic study at Wayne University is entitled Chemotherapy. It contains six lectures in this field by specialists actively engaged in the development of certain fundamental branches of chemistry. Volume I of the series was mentioned in these columns sometime ago.

The first lecture by William H. Feldman is entitled "Chemotherapy in Experimental Tuberculosis," covers chemotherapeutic agents, and the progress made in this field. Unexplored possibilities are pointed out and future developments depend largely on the acceptance of the fact that chemotherapy in tuberculosis is not a futile field. "Synthetic Antispasmodics" by Frederick F. Blicks is the next lecture and this is followed by "Chemistry of the Sulfa Drugs" by E. H. Northey. Harry S. Mosher discusses "The Antimalarial Problem"; C. Kenneth Banks covers "Organometallic Compounds as Chemotherapeutic Agents"; and Willard H. Wright treats on "Past Developments and Present Needs in the Chemotherapy of Parasitic Diseases" in the concluding papers. There is a subject index in the back of the book covering all papers.

R. H. OPPERMANN.

THE ENGINEER IN SOCIETY, by John Mills. 196 pages, 14 × 21 cms. New York, D. Van Nostrand Co., Inc., 1946. Price $2.50.

The subject of this book is not new. It has been discussed for at least two decades. Whenever a new scientific development became practical there was a revivial of discussion, until the most recent revival, a rather sudden and startling realization of dangers to civilization

and to the world itself of the power of science brought on by the atomic bomb. Science has progressed far ahead of economics and politics, for here is a tool, a magnificent development, powerful beyond ordinary conception for good or evil—and we do not have adequate machinery based on moral or legal principles for its use or control. Unless we can control the forces of politics and economics and direct their progress, in keeping with what is being accomplished scientifically, chaos is at our door.

The author of this book is experienced in scientific research, in dealing with scientists and in personal matters generally. It is his deduction that the engineer is capable of performing in the way of benefits to society through politics and economics of even greater magnitude than through his past scientific efforts. His trump card has not been played. Specifically it is the application of the scientific method, the idea of extending knowledge by quantitative experiments under controlled conditions where only one factor is allowed to vary at a time so that the experimenter can say: other things being equal X varies with A in a specific quantitative manner. The method, despite its proven power, is today so little employed outside of physical problems, is so diametrically opposed to the interests of many classes, that it cannot be applied in open attack. Its use on problems of economics and politics by scientists and engineers must be done quietly, away from the propaganda of vested interest. They then can come forth with their results, as they have for decades in their scientific conclusions. Only after they have completed their studies in an atmosphere of scientific investigation will it be time to present their findings. Then they can, and should, present the data they have obtained and their substantiation, their relationships, and the logical conclusions. This is the keynote of the work.

The book discusses practical and theoretical aspects of the problem of how to place the engineer in the position to accomplish this purpose. It covers ideas on aptitudes, broadening the perspective of the engineer, the relation of business executives to research, engineers' salaries, the licensing of engineers, and general attitudes of engineers. At times the reading does seem to stray somewhat from a path with defined borders but there are presented some challenging ideas which are thought provoking. The reader is led to the conclusion that he stands in awe of the possible results of scientific progress and that something must be done for society to catch up with it.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Charles Darwin and the Voyage of the Beagle, edited by Nora Barlow. 279 pages, illustrations, 14×21 cms. New York, The Philosphical Library, Inc., 1946. Price $3.75.

Communication Through the Ages, by Alfred Still. 201 pages, illustrations, 14×21 cms. New York, Murry Hill Books, Inc., 1946. Price $2.75.

Photography by Infrared, by Walter Clark. Second edition. 472 pages, illustrations, 14×22 cms. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Limited, 1946. Price $6.00.

Smith's College Chemistry, by Wm. F. Ehret. Sixth edition. 677 pages, illustrations, 18×25 cms. New York, D. Appleton-Century Company, 1946. Price $4.75.

Second Symposium on the Structure and Properties of the Diamond. 197 pages, illustrations, 18×24 cms. Reprinted from Proceedings of the Indian Academy of Sciences. Bangalore, The Indian Academy of Science, 1946.

CURRENT TOPICS.

Present Trends in Alloys.—Important progress has been made in the field of metallurgy during the war years which is being used to advantage in the present post-war period. Many new alloys have assumed their place in the metal industry, and as raw materials become more readily available it is expected that wider use of alloys throughout industry will result.

A broad series of standard triple alloy steels containing nickel, chromium and molybdenum, in which all of the significant alloying elements are under precise control, have become available to industry as a result of war-time experience. These steels accounted for an exceptionally large tonnage during the war and are still being employed in large quantities by users who find the controlled chemistries of such steels useful in assisting the maintenance of uniformity of their products. Of particular significance is the triple alloy steel tubing of the 8630 type which finds use in the aircraft industry because of its strength, ease of forming and ready weldability. Another outstanding development has been the application of 8650 steel for heavy duty springs. Hardenability, toughness and improved surface qualities of this steel have led to its growing favor.

There also has been considerable return to popularity of the standard pre-war alloy steels.

The demands for stainless steels continue to increase. It is estimated in trade circles that 1946 production will exceed or at least equal that of 1945. Of all stainless steel produced, the chromium-nickel types comprise more than two-thirds of the total.

While the demands for stainless by the aircraft industry have been drastically curtailed over the war period, it is predicted that this industry will continue to depend upon stainless steels in the airplanes of the future. The utilization of stainless steels, because of their strength and formability in addition to their resistance to corrosion by weather elements, continues unabated in the transportation industry for trains, trailers and other conveyances.

Other uses, showing continued satisfaction with the service records of stainless steels, include contractors' products (air conditioning, plumbing and heating), industrial equipment (chemical, food processing, textile, pulp and paper), household appliances (stoves, refrigerators, cooking utensils and tableware) and many others.

The need for materials to withstand the exceptionally high temperatures of jet and gas turbine engines has given great impetus to the development of so-called "super-alloys" for high temperature service, and as the demand for such engines becomes popular, a considerable market for these materials may be anticipated.

Industry is paying increasing attention to the engineering characteristics of alloy irons. Of note are the high strength gray cast irons containing $1\frac{1}{2}$ to 5% nickel, which are being used in Diesel engine crankshafts and cylinder liners and for aircraft piston rings. Strengths up to 100,000 lbs. per square inch are being obtained.

Modern high duty cast irons contain up to 38% nickel. A low expansion type of cast iron containing from 36 to 38% nickel is an important component in high precision machine tools. The abrasion-resisting martensitic cast irons containing 4½% nickel are rendering excellent service and are finding widespread use in steel mills, and the mining, cement, ceramic, power, paint and foundary industries.

A new cupro-nickel-aluminum bronze containing 12% nickel, 0.6% iron, 1.0% manganese and 1.5% aluminum combines superior resistance to salt water corrosion and excellent mechanical properties, including 85, to 95,000 lbs. per square inch tensile strength. It is expected to find broad usage in marine hardware and ship propellers where high strength and sea water corrosion-resistance are required.

There has been a marked expansion in the peace-time use of iron-nickel alloys possessing specialized thermal expansion properties as a result of more general use of automatic controls in electrically operated and temperature controlled products. The thermostatic bimetals ordinarily used to actuate these devices employ alloys containing 36 to 42% nickel for the low expansion side and a variety of alloy types containing 20 to 70% nickel for the high expansion side.

A marked increase also has occurred in the use of glass sealing alloys with 29 to 49% nickel for the manufacture of vacuum tubes, high wattage lamps, sealed beam headlights, etc. The unique magnetic characteristics shown by alloys of the group containing 47 to 84% nickel have become particularly important with the increasing use of higher frequency power applications in the electrical industry.

Nickel plating continues to be an increasing part of expanding platers' activity. Developments in high speed nickel plating baths, permitting the deposition of metal at higher current densities, have increased production capacities of shops with limited tank sizes and floor space. Bright nickel processes are being augmented by "semi-bright" deposits of good buffing qualities to reduce polishing and buffing costs.

A strong trend towards the use of nickel, Monel, Inconel and other high nickel alloys for consumer products has become widely evident. Several producers already have resumed the manufacture of Monel kitchen sinks. Corrosion resistant and rustproof hot water storage tanks for gas and electric heaters also are again being made of Monel.

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